Directional Effects of Load Deviations on the Buckling of Cylindrical Shells in Experiment and Design

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Abstract

Exploiting the lightweight design potential of thin-walled shells requires precise buckling load predictions. Due to numerous scattering parameters affecting the buckling load, probabilistic approaches are often used to model these effects. However, developing a stochastic model requires test data while making some simplifying assumptions. In this contribution, the influence of different load deviation types on the buckling load of axially loaded cylindrical CFRP shells is investigated. It is shown that effects are direction-dependent and vary between types. The extent to which probabilistic approaches account for such effects is discussed. Finally, the results are transferred to other load cases and the importance of considering directional effects in design and testing is highlighted.

Keywords

Buckling, Cylindrical Shells, Load Imperfections, Combined Loading, Probabilistic Design

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1. Motivation

To reduce material usage and achieve highly energy-efficient structures, lightweight design is an important aspect in the development of structural components, e.g., in the aerospace industry [1]. Thin-walled shells are one of the commonly used elements in aerospace as well as civil applications, e.g., in the fuselage of aircrafts and rockets and the load-carrying structure in offshore wind turbines [2], [3]. Particularly in the first case, reliability-based design criteria are a promising way to utilise the lightweight design potential of these shells [4]. Thin-walled shells exhibit failure through buckling under critical load cases such as axial compression, bending and torsion [5]. When the critical load is reached, this failure mode induces a sudden drop in the load-carrying capacity of the shells, potentially resulting in total failure of the entire structure in load-controlled applications. Due to the high buckling sensitivity, it is important to appropriately account for the factors that affect the buckling behaviour in order to achieve precise predicitions of the load-carrying capacity as well as realistic probability distributions. This currently poses a challenge in designing such structures, as existing design methods tend to deliver either overly conservative or, in some cases, non-conservative design loads [6], [7]. Probabilistic design approaches on the other hand usually require large amounts of empiric data while making some simplifying assumptions, the extent of which is dependent on the approach used [4]. In this contribution, the importance of accounting for directional effects and possible consequences of neglecting such behaviour are shown and the implications regarding the use of taylor-series based approaches in probabilistic design are discussed.

2. State of the art

2.1. Influences on the buckling load

The buckling load of thin-walled cylindrical shells is influenced by numerous factors [5], only few of which have been quantified to an extent that allows to reliably account for their influence. The most prominent and best characterised among these are geometric imperfections, which are defined as deviations from the perfect circular cylindrical shape of a shell [5], [8]. While geometric imperfections have been thoroughly investigated in the past, load imperfections are a known but not fully quantified influence that has been investigated in few studies, e.g. [6], [7] and [9]. In this contribution, load imperfections are defined as any deviation from perfect uniaxial and uniform load introduction [10], especially lateral forces, tilting of the shell and eccentric load introduction, which are visualised in Figure 1. Lateral loads, as recorded by Schillo in [11], occur if the main load vector is tilted with respect to the cylinder axis [12]. Tilting of the upper shell edge is usually the result of imperfections at the clamping or the interface to the test rig, as described in [6], [13]. Finally, eccentric load introduction occurs when a shell is not perfectly aligned with the test rig [14]. In several studies, e.g. [6], [14], it has been shown that the influence of such load imperfections changes based on the relative direction in the coordinate system of a shell. While for small magnitudes in sub-scale experiments the effects seem negligible, in practical application load deviation and combined loads are a regular occurrence, e.g., weight-induced axial load and wind-induced lateral loads in wind turbines [3].

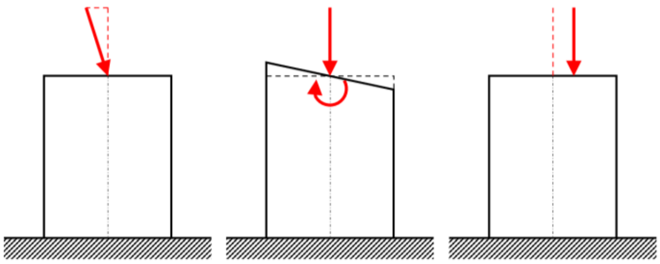

Figure 1: Schematic visualisation of three types of load imperfections occurring in axially compressed shells

2.2. Probabilistic design approaches

For the robust design of structural components, the uncertainties induced by the different influence factors, such as material, geometry and loading conditions need to be taken into consideration. Deterministic design approaches usually aim to find conservative design loads by using knockdown-factors [5] or global lower bounds of a system [15], thus avoiding the need for test or measurement data. Probabilistic approaches on the other hand attempt to model the different uncertainties and scattering parameters based on empirical data, while also relying on some simplifying assumptions when no sufficient database is available. One example for this are the aforementioned load imperfections. Whereas data on the amplitude of the imperfections can be gathered either directly from experiments [7] or by analysing the test results [6], the distribution of the direction they act in is highly influenced by the test rig [7]. Therefore, it is commonly assumed for load imperfections to follow a uniform distribution over the relevant portion of the circumference of a shell. Among probabilistic approaches, three different ways of incorporating load imperfections can be identified. Classical Monte-Carlo (MC) analyses assume a uniform distribution in the interval [-π, π), from which a large number of random samples is generated [4], [7]. General Taylor-series based approaches, like the First-Order-Second-Moment method (FOSM) or the Semi-Analytical Probabilistic procedure (SAP) [6] also consider the entire circumference. However, the effects of the load imperfections on the buckling load are only evaluated at three predefined points, as illustrated on the left side of Figure 2. Thirdly, some approaches like the Probabilistic Perturbation Load Approach (PPLA) [16] artificially create local imperfections at specific known points. As shown on the right side of Figure 2, when the location of the buckling inducing perturbation P_{Pert} is known, only load imperfections acting in the interval [-π/2, π/2] need to be considered.

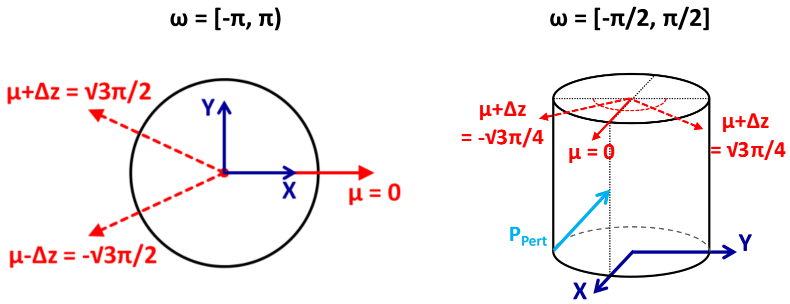

Figure 2: Schematic illustration of the evaluation of load imperfections in the SAP (left) and PPLA (right)

3. Research problem and objective

As outlined, the use of probabilistic approaches for the design of structural components generally requires some simplifying assumptions in regards to scattering parameters that have not been quantified. In this contribution, the challenges resulting from this practice are exemplified on the buckling of cylindrical shells, which are highly sensitive to scatter of the different influence factors. While the existence of load imperfections and their relevance is generally known, no viable quantitative description of the influence of different load imperfection types or the effects arising from the interaction of different imperfections are available. Therefore the question is posed, to what extent different types of load imperfections affect the buckling load of cylindrical shells under axial compression. Included herein is furthermore the investigation of possible directional effects. This in turn raises the question, how current design approaches, especially probabilistic ones, account for the different types of imperfections and in particular for direction-dependent behaviour. Finally, it is attempted to transfer the findings for uniaxial compression onto other load cases, including multiaxial loads, thus showing the importance of considering directional effects in testing and application as well as potential for generalisation of the results.

4. Approach and Methodology

In order to analyse the effects of different types of load imperfections, numerical investigations are carried out in a first step for the shells tested by Schillo et al. in [11] and by Hühne et al. in [15]. The dimensions and properties of the different shells, as well as references for the material data and imperfection patterns, are given in Table 1. In this contribution, the three types of load imperfections illustrated in Figure 1 are considered. An FE-model is set up in the same manner as described in [11] for all shells, incorporating the respective boundary conditions. For the shells Z07 - Z12 from [15], both shell edges are considered clamped while for Z1.1 - Z2.6 from [11], the upper edge is not constrained. As shown in [17], an exact representation of the existing boundary conditions is of major importance for the transferability of results between reality, test and simulation. If simplifications are made, the corresponding impact must be taken into account when analysing the results. Each shell is loaded with the three investigated types of load imperfection separately, before the axial load is increased until buckling occurs. The amplitude of the load imperfections is predefined in each case to be of realistic magnitude and is fixed over the course of the simulations to gain better comparability between results. For the shells tested by Schillo, the mean of measured lateral forces of 3.13 kN is used for the simulation, which is about 5% of the mean of buckling loads [11]. Based on this, a value of 0.93 kN is chosen for the lateral force acting on Z07 - Z12. The tilting angle is set to be 0.009° for all considered shells, based on the findings in [6]. No data on possible eccentricities are available for the tests of Hühne and Schillo. However, in a recent study by Takano et al., eccentric load introduction is applied in a controlled manner with amplitudes of roughly 1% to 1.6% of the shell radius [14]. Thus, an eccentricity of 1% of the radius is chosen, which corresponds to 2.5 mm and 1.14 mm for the shells from [15] and [11], respectively.

Table 1: Geometric and material properties of the investigated shells

reference	Hühne et al. [15]				Schillo et al. [11]
shells	Z07, Z08	Z09	Z10, Z11	Z12	Z1.1 – Z2.6
radius [mm]	250				114
free length [mm]	510				215
wall thickness [mm]	0.5				0.8
layup	[24,-24,41,-41]	[41,-41,24,-24]	[24,41,-41,-24]	[45,-45,0,-79]	[90,30,-30]s
material parameters	taken from [15]				taken from [7]
geometric imperfections	taken from [18]				taken from [19]

The results of the numerical analyses are then analysed with regard to the reduction in buckling load due to the different loading imperfections. Additionally, the scatter induced by varying the direction of the load imperfections is investigated and compared between the different types. Thirdly, different probabilistic approaches are carried out to determine buckling load distributions and design loads with and without accounting for the respective load imperfections. In [15], no data on the distribution of scattering parameters are given. Since the shells tested in [12] are of the same dimensions and have the same fibre orientations as Z07 – Z11, these ones are considered for probabilistic analyses. Consequently, the data for all scattering parameters are taken from [7] and [12] for the respective shells. The design approaches investigated herein are the previously described Probabilistic Perturbation Load Approach from [16], the Semi-Analytical Probabilistic procedure from [6] and design by using a classical frequentist Monte-Carlo analysis. The results are then compared with respect to how different ways of accounting for load imperfections influence the design load.

5. Results and Discussion

5.1. Numerical analysis of directional effects

Based on the available imperfection patterns and the predefined load imperfection amplitudes, the buckling loads are calculated for each case under variation of the active imperfection direction over the circumference in increments of 15°. The results of reference-calculations without load imperfections as well as the highest and lowest buckling loads achieved in each case are summarised in Table 2 for the shells from [15]. Generally, these shells exhibit a much higher sensitivity to load imperfections compared to the ones from [11]. Since the shells investigated by Schillo have a much lower ratio of radius to wall-thickness, they are expected to be less imperfection sensitive.

Table 2: Reference buckling loads and buckling loads under loading imperfections for shells from [15]

Shell	Z07	Z08	Z09	Z10	Z11	Z12
P_{Ref} [kN]	27.0	28.5	17.5	19.6	21.3	23.0
$P_{Tilt,min/max}$ [kN]	[24.3; 26.5]	[25.8; 27.0]	[15.8; 16.0]	[16.8; ***19.9***]	[18.5; 19.9]	[20.6; 21.3]
$P_{Lat,min/max}$ [kN]	[23.2; 25.3]	[24.5; 26.1]	[17.0; 17.1]	[17.3; 18.7]	[17.6; 19.3]	[22.6; 22.7]
$P_{Ecc,min/max}$ [kN]	[26.6; ***27.2***]	[28.0; ***28.7***]	[17.3; 17.4]	[19.4; ***19.7***]	[20.9; ***21.7***]	[22.7; 22.8]

For the shells Z1.1 – Z2.6 from [11], the results are given in Table 3. Here, the scatter induced by variation of the active direction ranges from 0.5% to 2.5% of the reference buckling load for all types of load imperfections. On average, the scatter resulting from tilting and lateral forces is slightly larger than the one from eccentric load introduction. At the same time, only the introduction of lateral forces yields a major decrease in buckling load of about 12% compared to the reference loads. Tilting and eccentric loading each only reduce the buckling load by about 2%. This significant difference in load reduction may well be a result of the chosen amplitudes for the different imperfections, a hypothesis that is supported by a parameter study regarding the magnitude of the eccentricity. As shown in Figure 3 exemplary for shell Z07, increasing the relative eccentricity to 5% of the shell radius results in a larger average reduction in buckling load while also showing a great increase in the direction-dependent scatter. However, misalignments between shell and test rig of 5 mm and more seem unrealistic compared to experience from experimental studies and thus are not considered any further [20].

Table 3: Reference buckling loads and buckling loads under loading imperfections for shells from [11]

Shell	Z1.1	Z1.2	Z1.3	Z1.4	Z1.5	Z1.6
P_{Ref} [kN]	65.0	65.0	65.0	64.9	65.0	64.8
$P_{Tilt,min/max}$ [kN]	[63.3; 64.4]	[63.3; 64.4]	[63.3; 64.5]	[63.8; 64.2]	[63.2; 64.4]	[63.2; 64.4]
$P_{Lat,min/max}$ [kN]	[56.4; 57.0]	[56.6; 57.2]	[56.8; 57.3]	[55.5; 57.0]	[56.3; 57.2]	[56.1; 57.1]
$P_{Ecc,min/max}$ [kN]	[63.9; 64.3]	[64.1; 64.4]	[64.0; 64.4]	[64.0; 64.3]	[64.0; 64.3]	[63.7; 64.4]

Shell	Z2.1	Z2.2	Z2.3	Z2.4	Z2.5	Z2.6
P_{Ref} [kN]	64.7	65.1	65.0	65.1	65.0	64.7
$P_{Tilt,min/max}$ [kN]	[63.2; 64.1]	[63.2; 64.2]	[63.7; 64.5]	[63.9; 64.4]	[63.5; 64.1]	[63.0; 64.0]
$P_{Lat,min/max}$ [kN]	[56.1; 57.3]	[56.7; 57.2]	[55.8; 57.0]	[56.3; 57.3]	[55.7; 57.0]	[55.5; 57.0]
$P_{Ecc,min/max}$ [kN]	[63.8; 64.3]	[64.0; 64.3]	[64.1; 64.4]	[64.1; 64.3]	[64.0; 64.3]	[63.6; 64.3]

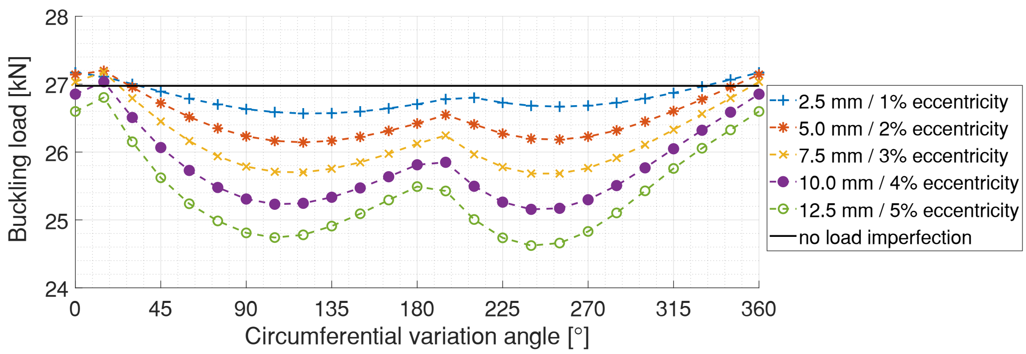

Figure 3: Buckling loads for Z07 from [15] under variation of eccentric load introduction

The highlighted buckling loads for each load imperfection type in Table 2 show that in some cases, the introduction of load imperfections can even increase the buckling load. This effect is also visible in Figure 3 and Figure 4. While it is apparent that sufficiently large magnitudes of load imperfections prevent such an increase in load-carrying capacity (see Figure 3), the results for Z10 in Figure 4 show that an increase in buckling load is possible under realistic load imperfections. In addition, the diagram illustrates that directional effects can greatly differ between imperfection types and, in some cases, are even inverted. While tilting of the shell yields the lowest buckling load at 240°, a lateral force in the same direction results in the highest load for that imperfection type. This relation however is not observed for all shells. Instead, in some cases two of the three load imperfection types exhibit a very similar directional influence. Consequently, it stands to reason that a qualitatively similar behaviour cannot be assumed for different load imperfection types in general, unless supported by appropriate investigations.

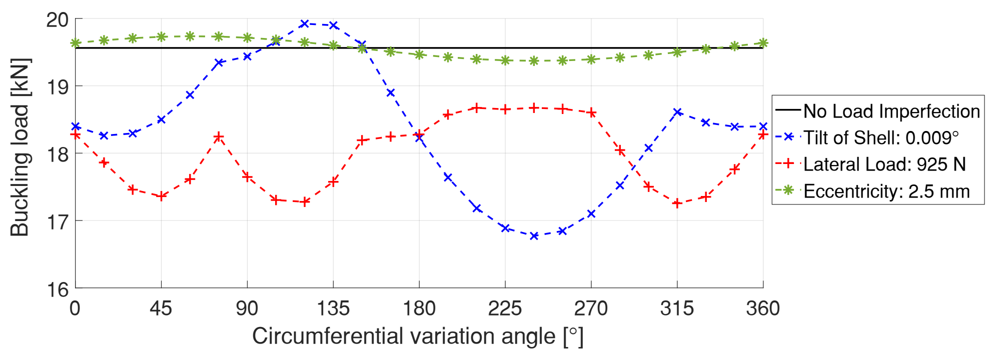

Figure 4: Buckling loads for Z10 from [15] under the effect of different load imperfection types

Based on the known influence factors on buckling, the source for the found directional effects in the influence of load imperfections is likely the interaction between local geometric and material imperfections and the deviation from perfect uniform load introduction [5], [16]. This is an important factor to consider in the setup of experiments and evaluation of test results, as well as during the design process. As shown, different types of load imperfections may result in qualitatively and quantitatively different directional behaviour. While in tests with uniaxial compression, the influence may be reduced to a minimum by an appropriate choice of boundary conditions and accurate modelling, new challenges arise when designing experiments with multiaxial loading. The industrial practice, both in aerospace and civil applications, shows that combined loads are a regular occurrence that needs to be considered to exploit the lightweight design potential. Following the findings for uniaxial compression in this section, it seems essential to reflect on the test set up, the procedure and the uncertainties induced by it, as some prevalent load cases, e.g. axial compression and bending [3], require purposefully non-uniform loading of the specimens.

5.2. Effects on probabilistic design

For the design using probabilistic approaches, the Monte-Carlo analyses are run with only 200 realisations, since the purpose of this study is to show a general behaviour rather than finding precise design loads in particular. For all probabilistic approaches, the target reliability of the shells is defined to be 99%. The design using a Monte-Carlo analysis and design with the PPLA are both carried out twice. In the first iteration, load imperfections are completely neglected while in the second, the respective types and values of load imperfections from the experiments are included. The taylor-series based SAP ist carried out for the considered shells in three iterations. In the first iteration, no load imperfections are considered, in order to gain reference values for design load and distribution. Then, two different descriptions of the load imperfection distribution over the circumference are used. A uniform distribution in the interval [-π, π) is assumed at first, followed by an iteration with the interval set to [0, 2π), resulting in a different value for the mean of the distribution while no actual change in probability density occurs. The resulting buckling load distributions, defined by mean μ_P and standard deviation σ_P, as well as the design loads P_{Design} are given in Table 4. All design loads that are not conservative with respect to the corresponding experimental results are highlighted.

Table 4: Buckling load distributions and design loads resulting from probabilistic design approaches

Shells	Load imperfection	$P_{Distribution}$ = $\mu_P \pm \sigma_P$ [kN]	P_{Design} [kN] (Reliability 0.99)
Schillo et al. [11]	MC, No load imperfection	65.6 ± 2.8	***59.0***
	MC, Lateral force, uniform distribution [0, 2π)	57.2 ± 3.3	49.5
	PPLA, No load imperfection	57.6 ± 2.6	54.2
	PPLA, Lateral force, uniform distribution [-π/2, π/2]	52.7 ± 3.4	48.4
	SAP, No load imperfection	65.6 ± 3.0	***58.5***
	SAP, Lateral force, uniform distribution [-π, π)	59.3 ± 3.6	51.0
	SAP, Lateral force, uniform distribution [0, 2π)	56.9 ± 3.6	48.6
Degenhardt et al. [12]	MC, No load imperfection	27.7 ± 2.1	***22.6***
	MC, tilting, uniform distribution [0, 2π)	23.3 ± 1.7	19.3
	PPLA, No load imperfection	18.4 ± 0.9	17.3
	PPLA, tilting, uniform distribution [-π/2, π/2]	16.9 ± 1.3	15.3
	SAP, No load imperfection	29.0 ± 2.9	***22.2***
	SAP, tilting, uniform distribution [-π, π)	24.6 ± 2.1	19.8
	SAP, tilting, uniform distribution [0, 2π)	24.0 ± 2.4	18.5

In all investigated cases, taking load imperfections into account leads to a significant reduction of design loads. For the Monte-Carlo analysis and SAP, the neglection of load imperfections leads to unconservative design loads even for the small sample sizes considered. The PPLA in turn, which is based on a lower-bound approach, delivers conservative results for the investigated shells both with and without accounting for load imperfections. As outlined by Meurer et al., the probability distribution resulting from the PPLA does not reflect the actual buckling load distribution of the shells and the true reliability of a given design is likely to be higher than the predefined value [16], thus yielding lower design loads. The inclusion of load imperfections further amplifies this effect, since their influence is evaluated only for the worst-case assumption that any deviation from uniform loading is acting in the direction of the failure-inducing local imperfection (see Figure 2).

In case of the SAP, the results in Table 4 show a significant difference between the two ways of describing the circumferential uniform distribution of the load imperfections. This points to one of the potential weaknesses of general taylor-series based approaches regarding robust design of lightweight structures. Since all scattering parameters are varied individually and the effects are combined through superposition, interactions between different influence factors, such as described in the previous section, may be neglected in some cases. Furthermore, determining the influence of load imperfections by calculating the buckling load for three predefined directions may produce skewed results that can still appear plausible. Considering only the two iterations that include load imperfections, the results of the SAP deliver a discrepancy of about 5% for the shells from [11] and about 7% for the shells from [15]. This effect is solely based on the mean imperfection pattern resulting from the available measurement data, on which the directional variation of load imperfections is imposed.

Considering that in a Monte-Carlo analysis all parameters are randomly determined for each realisation, no such effect is to be expected for this approach when using a sufficiently large number of realisations. These results show, in accordance with the findings of the numerical analyses presented in section 5.1, that the influence of randomly distributed load imperfections differs from other scattering factors, e.g., material parameters or wall-thickness. While, for example, an increase or decrease of stiffness parameters usually results in a correspondingly higher or lower buckling load, this elemental principle appears to not hold true for the circumferential direction of load imperfections.

5.3. Transfer to different load cases

In the following, it is shown that the findings for cylindrical shells under uniaxial compression can be transferred to different load cases and potentially to other geometries and applications. Here, the example of cylindrical shells under bending load is chosen, as this is one of the essential loads acting on offshore wind turbine towers [3] as well as one of the load cases that only few experimental studies exist for. FE-analyses are carried out for the previously investigated shells from Hühne et al. [15] and Schillo et al. [11] with variation of the bending direction along the entire circumference. Similar to the results presented earlier in section 5.1, the cylinders from Schillo's testing campaign are on average less sensitive to changes in the direction of the bending moment. The buckling loads for these shells exhibit a direction-dependent scatter of about 6% with loads ranging from about 3.8 kNm to 4.1 kNm. For the shells Z07 - Z12, different ranges for directional scatter are found. Z09 seems to be less sensitive with about 3% difference between highest and lowest achieved bending moment, while Z10 exhibits a direction-dependent scatter of about 16%. The results for this shell are visualised in Figure 5.

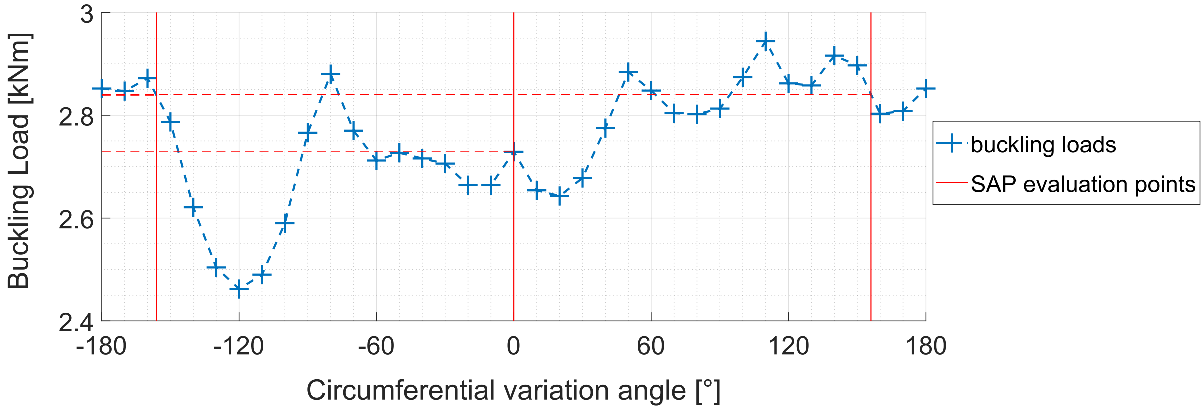

Figure 5: Buckling loads for Z10 from [15] under pure bending under variation of circumferential direction

The load case of pure bending is not dissimilar to the load imperfection type of tilting, which points towards the existence of the same challenge in regards to taylor-series based approaches. Since in taylor-series based probabilistic approaches, load imperfections are introduced to the mean of geometric imperfections, for the purpose of demonstration the imperfection pattern of Z10 can be considered to be such a mean imperfection. In Figure 5, the predefined directions for evaluation of the load influence when applying the SAP are marked for the classical assumption of a uniform distribution in the interval [-π, π). It is apparent that using this description of the distribution coincidentally does not account for the critical low buckling loads found at about -120° and thus the procedure yields an inaccurately low scatter and a higher design load. In practice, this may result in overestimating the true reliability of the structure. Considering the example of the load carrying structure of an offshore wind turbine, it is necessary to account for wind loads to act in all directions. Thus, the possibility of neglecting critical imperfections or imperfection modes, as shown for general taylor-series based probabilistic approaches, poses a potential source of unsafe designs.

6. Conclusion and Outlook

Reliability-based, probabilistic design approaches are a promising way to achieve a robust, lightweight design of structural components. In order to fully utilise the advantages of these approaches, it is essential to accurately account for scattering parameters of the considered system, both in the set up of a stochastic model as well as in experiments to generate the necessary database. While the influence of some factors, e.g., material parameters, can usually be approximated by linearisation, it is shown that for others, like load deviations, the influence varies, depending on the effective direction. In this contribution, the direction-dependent impact of different types of load imperfections on the buckling of cylindrical shells was investigated by way of example. It was found that different types of load imperfections do not necessarily cause qualitatively or quantitatively similar behaviour. Thus, the importance of considering load imperfections when designing and conducting experiments is outlined, particularly in case of multiaxial testing where non-uniform loads are purposefully introduced.

Furthermore, it is shown that the application of general taylor-series based probabilistic approaches to design challenges with direction-dependent effects may lead to unconservative results, depending on the assumptions made for certain distributions. Thus, it is recommended to first investigate the strength and scatter of direction-dependent effects when the occurrence of such behaviour cannot be ruled out, before choosing suitable methods for probabilistic analyses.

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