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FAIRNESS AND MANIPULATION: AN EMPIRICAL STUDY OF ARROW'S IMPOSSIBILITY THEOREM

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Abstract

The design process often requires work by teams, rather than individuals. During these times it is likely that situations will arise in which members of a team have different opinions, yet a group decision must still be made. Unfortunately, Arrow's Impossibility Theorem indicates that there is no method for aggregating group preferences that will always satisfy a small number of "fair" conditions. This work seeks to identify methods of combining individual preferences that can come close to satisfying Arrow's conditions. Experiential conjoint analysis was used to obtain empirical utility functions for drinking mug designs. A number of functions for constructing group preference were then analysed using both empirical conjoint preferences and randomly generated preferences. This analysis involved checking each of Arrow's conditions, as well as computing the likelihood that a method will be susceptible to manipulation by a dishonest individual. Based on the results, methods that should be used to aggregate group preference in practice are recommended.

Keywords: Decision making, Teamwork, Collaborative Design, Group preferences

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1 INTRODUCTION

Much of the design process is the result of teams, rather than individuals (Paulus et al., 2011). During team-based design, there often arise situations in which members of a team have different opinions, yet a group decision must still be made. Unfortunately, a proof by Arrow (1950) indicates that there is no method for aggregating group preferences that will always satisfy a small number of "fair" conditions. Previous work has examined Arrow's theorem and its application to design through a theoretical approach (Hazelrigg, 1996; Scott and Antonsson, 1999). The work presented in this paper uses an empirical approach, which attempts to identify methods for aggregating individual preferences that have a high likelihood of being fair, according to the conditions put forth by Arrow. In addition, it is often possible for a sole individual to manipulate the outcome of the aggregation procedure if they have sufficient knowledge of their teammates' preferences. For that reason, this work also examines the susceptibility of preference aggregation methods to manipulation.

The empirical individual preferences used in this work were measured through conjoint analysis. Conjoint analysis has been widely applied in engineering design problems due to its ability to mathematically capture preference using a utility function (Luce and Tukey, 1964; Green, 1974; Green and Wind, 1975). In conjoint analysis, data are typically collected through a survey in which participants are asked to rate, rank or choose between different offerings composed of varying combinations of researcher specified product attributes. Each attribute is described by multiple levels, which represent the variability in the parameter being investigated. Levels of different attributes are varied based on standard design of experiment techniques. Based on participant response data, preference weights can be determined for each attribute and level that was tested during the experiment. A mathematical representation of preference can then be created to describe preference for every design within the design space under investigation.

The representations of the product attributes in conjoint analysis were traditionally limited to descriptive text. However, recent developments have expanded upon conjoint analysis by utilizing more complex forms of attributes. Orsborn et al. (2009) introduced an extension of conjoint analysis, termed continuous visual conjoint analysis, which can derive utility functions based upon preference for continuous aesthetic attributes (seen in 2D in that work), resulting in preference that can be extrapolated to any point within the continuous design space explored. Several researchers have further explored visual conjoint methods (see examples such as Sylcott et al., 2013 and Kelly et al., 2007). Beyond visual conjoint analysis, Tovares et al. (2014) developed experiential conjoint analysis based upon experience based preference judgments (touching, manipulating, etc.), where again preference could be extrapolated.

Once individual preferences are known, it is often necessary to develop a unified group preference that considers the preferences of a number of individuals. Functions that create group preference structures are generally referred to as aggregation functions. These are simply functions that take as input a set of individual rankings, and return a single, group ranking. An important issue when considering any aggregation function is manipulability. An aggregation function is manipulable if an individual can achieve a more preferred group ranking by misreporting their own preferences. Unfortunately, every deterministic aggregation function is manipulable in at least some circumstances (Gibbard, 1973; Satterthwaite, 1975). Further, complete information of all individuals' preferences is necessary to compute a dependable manipulation (Bartholdi et al., 1989). Design teams tend to be composed of a small number of individuals that are familiar with one another's preferences (Wegner, 1986). This makes it probable that an individual would be capable of collecting the information necessary to effectively manipulate a group decision scenario.

Even if manipulability is disregarded, the aggregation of individual preferences into a group ranking is still non-trivial. Consider three individuals, who must form a group ranking over three alternatives (A, B, and C). Their set of individual preferences, also known as a preference profile is as follows. Individual 1 has the ranking A $\succ$ B $\succ$ C, individual 2 has the ranking B $\succ$ C $\succ$ A, and individual 3 has the ranking C $\succ$ A $\succ$ B. This specific preference profile is commonly known as the Condorcet paradox (de Condorcet, 1785). One method that can be used to develop the required group ranking is the pairwise majority rule. The pairwise majority rule would be implemented as follows: a majority of voters prefer *A* to *B*, therefore the group should also prefer *A* to *B*. A majority of voters also prefer *B* to *C*, so the group should also reflect this preference. Finally, a majority of voters prefer *C* to *A*, so the group should prefer *C* to *A*. In summary, the group should prefer *A* to *B*, *B* to *C*, and *C* to *A*. This is an irrational and cyclic group preference, and provides no basis upon which to make a decision. Motivated by this paradox, Arrow (1950) proved that no aggregation function can always satisfy a small set of reasonable conditions. The conditions constituting Arrow's theorem are stated as follows (Arrow, 1950; Nissan, 2007):

1. **Unrestricted Domain**: The aggregation function is defined for all preference profiles, for any number of voters and any number of alternatives.
2. **Unanimity**: If all individuals prefer x to y, then the group ranking must also prefer x to y.
3. I**ndependence of Irrelevant Alternatives**: The group preference between alternatives x and y must depend solely on individual preferences between x and y.
4. **Citizen Sovereignty**: There exists a preference profile that can make any alternative a winner.
5. **Non-Dictatorship**: The aggregation function does not simply return a specific individual's ranking.

The Independence of Irrelevant Alternatives (IIA) condition is often criticized as being overly restrictive (Luca and Raiffa, 1957). Less restrictive versions of this IIA condition have been proposed. For instance, Young (1985) proposed Local Independence of Irrelevant Alternatives (LIIA), which only considers the removal of the first and last candidates in the group ranking. Despite such criticism, other work has demonstrated results similar to Arrow's that do not depend upon IIA (Seidenfeld et al., 1989).

The implications of Arrow's Theorem for design have become a subject of debate. Hazelrigg (1996) discussed the relationship of Arrow's Theorem to optimal design, and concluded that commonly used methods like Total Quality Management and Quality Function Deployment may provide results that do not accurately reflect users' preferences. Scott and Antonsson (1999) argued that, although group decisions must be made in design, the multi-criteria aspect of many design decisions allows group preference to be determined without any conflict with Arrow's Theorem. The current work proposes a pragmatic view of Arrow's theorem within the context of design. While there are cases in which design decisions are unambiguous (Scott and Antonsson, 1999), there are also situations in which varying opinions between individuals are pivotal (e.g. early conceptual design). For such situations, Arrow's theorem states that there is no procedure for creating a group preference that will *always* offer fair results.

This work examines Arrow's theorem from an empirical point of view. Experiential conjoint analysis is used to query real preferences for a class of products. Using these empirical preferences, simulated voting scenarios are constructed and analysed to determine the extent to which several aggregation functions satisfy Arrow's conditions. Randomly generated individual preference profiles are also explored to provide a baseline against which to compare the collected empirical data. The probability of manipulation by a dishonest individual is also evaluated within the simulated voting scenarios. Finally, the aggregation function that is most likely to provide results that are strategy-proof and fair (in accordance with Arrow's theorem) is proposed.

2 METHODS

This work employs a three-step approach combining user studies and computational modelling. First, individual preferences for different variations of a parameterized drinking mug were measured through experiential conjoint methodology. Second, the results of the conjoint study were used to generate a distribution of personal utility functions. Finally, this distribution was used to simulate the performance of five aggregation functions. These functions were analysed to assess how often they fulfilled Arrow's conditions, and how often they could be manipulated.

2.1 Experiential Conjoint Study

For this analysis, 3D printed ceramic drinking mugs were used as a stimulus to determine individual preferences (Tovares et al., 2014). The mugs were 3D printed in accordance with a pre-defined experimental design. Three continuous attributes, each represented by three levels, were chosen to decompose the product: height (75mm, 95mm, 115mm), base width (40mm, 60mm, 80mm), and handle curvature (three Bezier curves, each defined by 3 points). The three levels that were chosen to describe the drinking mug design space, created a space containing 27 (3^3) candidate designs.

Participants for the empirical portion of the experiment were recruited through two undergraduate courses at Carnegie Mellon University, and were compensated with course credit for their

participation. In total, 51 participants completed the 25-minute study, which was conducted in two parts.
In the first part, each participant was asked to independently rate 17 ceramic drinking mugs. The drinking mugs were rated on a scale from 1 (least appealing) to 10 (most appealing). Participants were presented with one of two random orders of drinking mugs. Data from 15 participants were omitted due to failure to meet the minimum accuracy requirements enforced through the duplicate rating task in the experiment.
In the second part of the study, participants were asked to perform a ranking task. Participants were asked to independently rank 4 drinking mug designs in order from most appealing to least appealing. The designs chosen for this task are shown in Figure 1. These specific designs were chosen to represent significant variety across the three attributes. This allowed the participants' explicit ranking to be compared to the ranking predicted through conjoint analysis.

Figure 1. Drinking mugs used for ranking task (from left to right: A, B, C and D).

2.2 Generating Preference Profiles

Experiential conjoint analysis was used to build utility functions for every study participant. In this work, a 9 question, balanced and orthogonal model was used to estimate the preference model parameters. A Gaussian distribution was then used to describe the parameters of these individualized utility functions, and build unique, *empirical* preference profiles. Drawing from an empirically developed distribution produces preference combinations that are likely to occur in reality. Merely generating all possible preference combinations, or building preference profiles from random orderings, would have no such link to real preferences.
Using the ratings data collected from study participants, the attribute preference weights, $\boldsymbol{\alpha}$, were solved for using ordinary least squares regression:

$$\alpha = \left(\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{y}. \tag{1}$$

Here, **X** is the coded design matrix, and **y** is the vector containing ratings for each design alternative provided by the survey participants. Taking advantage of the continuous nature of the attributes employed in this study, differentiable quadratic utility functions are used to model preference. Quadratic utility functions were chosen to allow for maxima that were not corner solutions, and to remain consistent with prior work (Tovares et al., 2014; Orsborn et al., 2009). The form of this model is shown in Equation 2:

$$u_{r,q} = \alpha_0 + \sum_{i=1}^{p}\left(\alpha_{1,i}\boldsymbol{X}_i + \alpha_{2,i}\boldsymbol{X}_i^2\right). \tag{2}$$

The variable $u_{r,q}$ denotes the total utility, *u,* for the r^{th} participant, and q^{th} design. The calculation of the total utility requires the uncoded design matrix, **X**, and the individual attribute preference weights, $\boldsymbol{\alpha}$. The quality of each preference model was validated using the mean absolute error criterion.
After calculating preference weights for each individual using Equation 1, a Gaussian distribution was defined to describe the distribution of preference weights. This distribution over the values of α can be summarized by the mean vector and covariance matrix of the sample of individual values of α. To generate a preference profile, a vector is drawn from the α-distribution. This vector represents the unique utility function for an individual, with a functional form as provided in Equation 2. The utility function is then used to calculate a utility for each of *m* randomly selected design alternatives. Using these utilities, the alternatives are ranked in order of decreasing utility, and this ranking is added to the preference profile. The process of drawing vectors, creating utility functions, and ranking alternatives is repeated *n* times, thus building an *n*-individual/*m*-alternative preference profile. This methodology

enables the construction of empirical preference profiles with any number of individuals (n) and any number of design alternatives (m).

2.3 Aggregation functions, Arrow's Conditions, and Manipulability

Aggregation functions provide a method for building a group ranking from a preference profile. For an aggregation function and a given preference profile, Arrow's conditions can be evaluated, and whether or not the group ranking is manipulable can be determined. By performing this analysis for many different preference profiles and the same aggregation function, it becomes possible to estimate the probability that the function will satisfy Arrow's conditions, and the probability that the result can be manipulated.

2.3.1 Aggregation functions

Five aggregation functions were evaluated as part of this work. These included three positional scoring functions (plurality, veto, and Borda), and two multi-step functions (Instant Runoff Voting and Copeland). These functions were selected because they are well-studied in the voting theory literature, and offer variety in terms of the information that must be provided by individuals, and the complexity of computing a group ranking. A positional scoring rule is defined by a scoring vector s of length m, where m is the number of alternatives. Each voter allots s_k points to their k^{th} most-preferred alternative. To establish a group ranking, the number of points scored by each individual is counted. The group ranking is simply a ranking of alternatives in order of most points scored. The scoring vectors for the plurality, veto, and Borda functions are $[1,0,\ldots,0,0]$, $[1,1,\ldots,1,0]$, and $[m-1, m-2,\ldots,1,0]$, respectively.

The two multi-step aggregation functions used in this work both use the plurality function. The Instant Runoff Voting (IRV) function is composed of m rounds. In each round, the plurality function is applied, and the alternative with the least points is removed from the alternative set. The next round begins with the updated set of alternatives. This continues until only a single alternative remains. The group ranking is defined by the order in which alternatives were removed from contention. The Copeland aggregation function performs a plurality vote between every pair of alternatives. For every pairwise election that an alternative wins, it receives one point. For every loss, it loses one point. The group preference is then a ranking of alternatives in order of net points earned.

2.3.2 Analysis of Arrow's Conditions and Manipulability

Let a *preference scenario* be a combination of a specific preference profile and an aggregation function. The aggregation function uses the preference profile to produce a group ranking. For any preference scenario, it is possible to check whether or not Arrow's conditions are satisfied. The Unrestricted Domain condition will be addressed by generating profiles with differing numbers of alternatives and team members. The Non-Dictatorship and Citizen-Sovereignty conditions are dependent only on the aggregation function, and are satisfied by the aggregation functions chosen for this work. The Unanimity and IIA conditions are dependent upon the specific preference scenario. The unanimity condition was checked by first finding the pairwise preferences that were shared by all individuals. If these unanimous preferences were also found in the group preference, then the preference scenario satisfied the condition. To assess the IIA condition, the group ranking for the preference profile was first computed. Then, an alternative was either added to or removed from the set of alternatives. The preference profile was updated according to individuals' utility functions, and a new group ranking was computed. If the relative position of original (or remaining) alternatives in the new group ranking was unchanged from that in the original ranking, then the preference scenario satisfied the IIA condition.

Further, we define a concept called *Conditional Arrow Fairness.* A preference scenario exhibits Conditionally Arrow Fairness if it satisfies the Unanimity and IIA conditions, and if the aggregation function is non-dictatorial. This concept is conditional upon Arrow's first condition (Unrestricted Domain) because it is checked using preference profiles with a specific number of alternatives and individuals. However, by generating and checking many profiles, the probability that a given aggregation function will be Conditionally Arrow Fair can be established. This serves as an indication of the performance of aggregation functions in their ability to come close to satisfying Arrow's conditions.

Perfect knowledge of the preferences of all individuals is often necessary to compute a dependable manipulation (Bartholdi et al., 1989). Design teams tend to be composed of a small number of individuals, and individuals who work closely as part of a team can become familiar with one another's preferences (Wegner, 1986). If an individual develops sufficient familiarity with their teammates' preferences, manipulation becomes a real possibility. Therefore, every preference scenario was assessed to determine if a single individual could manipulate the scenario. This assessment was accomplished by modifying the preferences of the manipulator until a successful manipulation was found, or until all possible preferences were attempted. A successful manipulation is an individual preference modification that would result in a more preferred group ranking for the manipulating individual. This process was repeated for every individual as the manipulator. If a successful manipulation was not found, then the preference scenario was deemed to be strategy-proof.

3 RESULTS

Before presenting the results of the empirical simulations, the results of simulations using random preference profiles will be provided. Random preference profiles are a worst-case scenario for the formation of group preference, because a random preference profile is likely to show more variance in preferences than what would be observed from real-world data. This provides a good basis for comparison to the empirical results. For both random and empirical preference profiles, aggregation functions are compared using preference profiles with varying numbers of individuals (from 3 to 15) and alternatives (from 3 to 6).

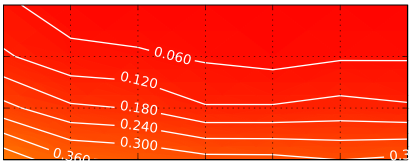
3.1 Random Preference Profiles

In this section of the analysis, all preference profiles were composed of randomly generated rankings of design alternatives (with no input from the conjoint analysis). Randomly generated preferences represent a worst-case scenario for the formation of a group ranking since it is unlikely that there will exist any tacit agreement. Conditional Arrow Fairness and strategy-proofness were evaluated using 1000 random preference profiles for every combination of number of individuals (from 3 to 15) and number of alternatives (from 3 to 6). Table 1 shows the results for this analysis, averaged across all preference profiles.

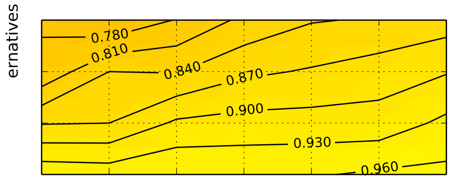
Table 1. Average results for random individuals.

Aggregation function	Strategy-proof	Unanimity	IIA	Conditional Arrow Fairness
Plurality	80.9%	97.1%	1.2%	1.2%
Veto	74.0%	97.2%	1.2%	1.2%
Borda	71.6%	100.0%	11.8%	11.8%
IRV	87.0%	97.4%	1.9%	1.9%
Copeland	88.1%	100.0%	15.1%	15.1%

Strategy-proofness ranges from approximately 70% for the Borda function to nearly 90% for the Copeland function. Conditional Arrow Fairness is hard to achieve with random preference profiles; the Borda function and Copeland function have probabilities of Conditional Arrow Fairness that exceed 10%, but every other function falls below 2%. The Copeland function achieves both the highest probability of Conditional Arrow Fairness *and* the highest probability of strategy-proofness. Figure 2 shows the dependence of the Copeland function's characteristics on the number of individuals and the number of alternatives in the preference profile. The contours indicate either the probability of Conditional Arrow Fairness (Figure 2(a)), or the probability of strategy-proofness (Figure 2(b)). For every grid point in the plot, 1000 random preference profiles were created and analysed.



(a) *Probability of Conditional Arrow Fairness*



(b) *Probability of strategy-proofness*

Figure 2. Copeland aggregation function characteristics.

An examination of the contour plots in Figure 2 indicates that the probabilities of both Conditional Arrow Fairness and strategy-proofness can be increased by decreasing the number of alternatives. Furthermore, despite the fact that Copeland is fairly strategy-proof for most cases, the highest probability of Conditional Arrow Fairness that it can obtain is at best slightly greater than 30% in this random case.

3.2 Empirical Preference Profiles

Here, results are presented that depend upon the empirical data generated from the experiential conjoint survey. Data from 15 participants were omitted due to failure to meet the minimum accuracy requirements for this duplicate task. The MAE of the remaining participants was calculated to ensure that the model predicted accurate ratings for the survey respondents. The mean model MAE was 1.17±0.59, which is commensurate with the MAE of the experiential conjoint model created by Tovares et al (2014).

Recall that the conjoint analysis was used to create a probability distribution of utility functions, and that this distribution was used to create unique empirical preference profiles. Therefore, some preference relations will be much more probable than others, resulting in preference profiles that are likely to show some level of tacit agreement between individuals. Conditional Arrow Fairness and strategy-proofness were evaluated using 1000 empirically generated preference profiles for every combination of number of individuals (from 3 to 15) and number of alternatives (from 3 to 6). Table 2 shows the average results of this analysis, averaged across all preference profiles.

Table 2. Average results for empirical individuals.

Aggregation function	Strategy-proof	Unanimity	IIA	Conditional Arrow Fairness
Plurality	94.6%	87.5%	3.9%	3.9%
Veto	56.5%	91.9%	4.3%	4.3%
Borda	77.3%	100.0%	39.1%	39.1%
IRV	97.1%	87.4%	4.5%	4.5%
Copeland	97.5%	100.0%	66.6%	66.6%

The IRV and Copeland aggregation functions are strategy-proof (not manipulable by a single individual) in more than 95% or preference profiles. The veto rule is the worst, since it allows nearly half of all preference profiles to be manipulated by an individual. Conditional Arrow Fairness is an even starker criterion for differentiating the aggregation functions. Plurality, veto, and IRV provide Conditional Arrow Fairness in less than 5% of preference profiles. Borda is slightly better at approximately 40%, and Copeland is the best, providing Conditional Arrow Fairness in more than 65% of preference profiles. For these criteria, Copeland is once again clearly better than the other aggregation functions. Figure 3 shows the dependence of the Copeland function's characteristics on the number of individuals and the number of alternatives in the preference profile. The contours indicate the probability of Conditional Arrow Fairness (Figure 3(a)), and the probability of strategy-proofness (Figure 3(b)). Once again, every grid point represents the average of 1000 simulated preference scenarios.

(a) Probability of Conditional Arrow Fairness

(b) Probability of strategy-proofness

Figure 3. Copeland aggregation function characteristics.

The probability of Conditional Arrow Fairness, shown in Figure 3(a), appears to be primarily a function of the number of alternatives (with fewer alternatives resulting in a higher probability). The probability of strategy-proofness, shown in Figure 3(b), is a function of both number of individuals and number of alternatives. The probability of strategy-proofness appears to asymptotically approach 100% for a large number of individuals and a small number of alternatives. It should be noted that the Copeland function is well above 90% strategy-proof for most of the preference profiles explored in this analysis.

The results that have been discussed thus far are predicated upon the utility functions developed through ratings-based conjoint analysis. For that reason, participants in the conjoint study were also asked to explicitly rank a subset of four designs (the same four designs shown in Figure 1). This allows the comparison of the group ranking predicted by the utility functions to be compared to the group ranking computed directly from rankings provided by study participants. Table 3 shows the group preferences predicted from the conjoint utility functions, the group preferences computed directly from the ranking task data, and the Kendall's Tau statistic relating the two rankings. Any differences between the two aggregate rankings are underlined. Most aggregation functions (Plurality, Borda, IRV and Copeland), return an aggregate utility-based ranking that is in perfect agreement with the aggregate empirical ranking. The sole exception is the Veto aggregation function, which displays disagreement amongst the top two alternatives. The Veto aggregation function is the only function explored in this work that directly counts votes *against* the least preferred alternatives. All other functions count, in some way, votes that *support* various alternatives. Therefore, this result could indicate that voting in support of design alternatives more firmly resolves a group preference structure.

Table 3. Comparison of group preference from utility function and ranking data.

Aggregation function	Utility Function Group Preference	Ranking Data Group Preference	Kendall's τ Correlation Coefficient
Plurality, Borda, IRV and Copeland	A ≻ B ≻ C ≻ D	A ≻ B ≻ C ≻ D	1.000
Veto	B ≻ A ≻ B ≻ D	A ≻ B ≻ C ≻ D	0.667

4 DISCUSSION

This work used both empirical preference profiles (generated from experiential conjoint study results) and uniform random preference profiles. Uniform random preference profiles serve as a worst-case scenario for the formation of group preference, because it is unlikely that individuals will display agreement. In more realistic preference profiles, it is likely that individuals will agree on at least some preference judgements. And it a segment is properly created then alignment should become even more consistent. A detailed analysis using both random and empirical preference profiles was performed with 8 individuals and 5 alternatives (Tables 1 and 2). For both uniform random and empirical preference profiles, the Copeland aggregation function displayed the highest probability of Conditional Arrow Fairness, and of being strategy-proof. For some empirical profiles, the probability of Conditional Arrow Fairness can exceed 80%, and the probability of strategy-proofness exceeds 98% (see Figure 3). However, for uniform random preference profiles, the probability that Copeland will *fail* Conditional Arrow Fairness can exceed 90% (see Figure 2(a)). This motivates a discussion of the practical implications of a failure of Conditional Arrow Fairness, which may result from a failure of either IIA or Unanimity.

A failure of Unanimity indicates that all individuals in a preference profile ranked *x* over *y*, but the group ranking did not. This is not always an egregious fault. Assume that a group is trying to select their most preferred alternative from the set {*a*, *b*, *c*, *x*, *y*} and all members of a group prefer alternate *x* to alternative *y*. If the final group ranking is $a \succ b \succ c \succ y \succ x$, the failure of Unanimity is relatively harmless. However, if the final group ranking is $y \succ x \succ a \succ b \succ c$, the failure of Unanimity is much more serious. A failure of IIA is more likely than a failure of Unanimity (see Tables 1 and 2). IIA failure means that adding (or removing) an alternative from the preference profile changes the relative ranking of the original (or remaining) alternatives. Consider a situation in which the group ranking is $a \succ b \succ c \succ d$, but the addition of alternative *x* changes the group ranking to $a \succ b \succ d \succ x \succ c$. The relative ranking of alternative *c* and *d* changed. If the purpose of the construction of the group ranking is to select the most preferred alternative, then this failure of IIA is inconsequential. However, if the purpose of the ranking is to eliminate the least preferred alternative, the result is more troublesome. These examples illustrate the fact that the importance of any failure of Conditional Arrow Fairness is highly context-dependent. Utilizing an aggregation function that has a high probability of Conditional Arrow Fairness provides protection against both trivial and serious failures of these conditions. By structuring decisions so that the number of individuals is much larger than the number of alternatives, the Copeland function can achieve a high probability of Conditional Arrow Fairness (over 80% in this case), thus protecting against failures of IIA and Unanimity in the majority of situations.

Often, preference data from user surveys must be aggregated into a single group preference before use in design. According to this study, the application of the Copeland aggregation function to this task would maximize the likelihood that the resulting group ranking has fair characteristics (as defined by Arrow's conditions). Our results also indicate that keeping the number of alternatives in such tasks small (relative to the number of participants) increases the probability of a developing a fair group ranking.

The results of this work are sufficiently general that they have potential be applied to domains other than the aggregation of user data. For instance, it is often necessary for design teams to rank design alternatives, usually to narrow down the number of alternatives before continuing work. If the quality of the designs is not readily quantifiable, preference over design alternatives becomes a matter of personal opinion. Therefore, individuals' rankings over design alternatives may vary enough that the group ranking isn't immediately obvious. Applying the Copeland function in this situation would allow the team to form a group ranking that is more likely to have fair characteristics. The result would also have a higher likelihood of strategy-proofness, meaning that team members would have no incentive to provide anything but their true ranking of the alternatives.

5 CONCLUSIONS

This work took an empirical approach to examine several methods for combining individual preferences into a group preference. Each of these methods, commonly referred to in this work as aggregation functions, was analysed in terms of manipulability and Conditional Arrow Fairness. Of the aggregation functions explored in this work, the Copeland function offers the highest probability of Conditional Arrow Fairness and the highest probability of strategy-proofness. This indicates that it is likely to return a fair result, and that individuals would thus have no incentive to provide anything but their true preference for the alternatives. The Copeland function could be applied to a variety of domains, including the aggregation of preferences from user survey and decision-making during the design process.

Future work should extend this analysis to a larger set of aggregation functions, and explore the use of the Copeland function in more complex and longitudinal design contexts. In addition, the effort required from an individual to quickly and accurately compute a manipulation should be examined.

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