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TOPOGRAPHIC OPTIMIZATION WITH VARIABLE BOUNDARY CONDITIONS: ENABLING OPTIMAL DESIGN OF INTERACTING COMPONENTS

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ABSTRACT

Topographic optimization provides a valuable opportunity for the design of optimal structures. A significant limitation in the current generation of topographic optimization algorithms is the non-inclusion of boundary conditions as optimization variables. This limitation significantly constrains the domain of design problems compatible with topographic optimization. For example, unique components can be optimized for a given set of boundary conditions only. There is no opportunity to assess whether these boundary conditions are themselves optimal. This work reports on the authors novel contributions to allow boundary conditions to be included as optimization variables, thereby dramatically expanding the domain of design problems that are compatible with topographic optimization. This method is demonstrated by the optimal topographic optimization of interacting components: a previously intractable design problem.

Keywords: biomimetics, design for X, finite element method, topographic optimization

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1 INTRODUCTION

Topographic optimization (TOP) refers to the search for optimal geometry for a scenario defined by objectives, such as minimal mass or cost; and, subject to constraints, such as allowable spatial envelope or maximum allowable stress. Topographic optimization results in an optimal material distribution subject to these initial conditions. As such, topographic optimization is not based on *a priori* assumptions of material distribution, and consequently provides a significant opportunity for innovative structural design. However, the efficacy of topographic optimization is limited in practice by the non-inclusion of boundary conditions as optimization variables. This limitation constrains the scope of valid optimization problems to allow only geometric optimization based on the *a priori* assumptions of boundary conditions – assumptions that are not necessarily optimal.

By review of current TOP implementations reported in the literature, the authors have developed a novel TOP algorithm that enables enhanced outcomes in the field of topographic optimization by including boundary conditions in the associated solution. This outcome extends the domain of valid problems to allow the optimization of interacting components. This previously unavailable capability is of significance to holistic design as it allows the design of interacting components that are optimal at the systems level.

2 TOPOGRAPHIC OPTIMIZATION

Topographic optimization is mathematically defined as the search for optima associated with objective functions subject to a series of identified constraints (Brackett 2011).

The objective functions may be either defined explicitly as an algebraic function, or implicitly within a *black box* discrete system. Common objectives include the minimization of:

- System mass
- System cost
- Deflection
- Thermal resistance (or conductivity)
- Energy absorption

Typical design constraints include:

- Allowable design space
- Allowable component deflection
- Allowable material stress
- Boundary conditions such as loading, symmetry and physical constraints

Topological optimization is a multidisciplinary field of study that has been applied to a range of structural problems; including the biomimicry of natural light weight structures (Deshpande, Fleck *et al*., 2001), and the structural optimization of Passion Facade of the Sagrada Familia (Xie 2011). In early works, including pioneering work of Mitchell (1904), an analytical approach was used to solve the optimization problem. In recent years researchers have incorporated numerical CAE tools and computer algorithms to increase the domain of feasible TOP problems. For example, Xie and Steven (1993) used numerical simulation tools to optimize 2D geometry subject to a condition of maximum allowable stress. Mattheck and Burkhardt (1990) proposed a new method called Computer Aided Optimization (CAO) to mimic natural structures. This method defines stiffness as the design variable, as opposed to stress as used in previous methods. Sigmund (2001) used a highly efficient algorithm to further simplify the automation of topology optimization. Huang and Xie (2009) introduced use of penalization number in Bi-directional Evolutionary Structural Optimisation (BESO) which enabled the use of soft material instead of creating voids in the structure to obtain stiffness optimization.

2.1 Structural optimization approaches

Topographic optimization is approached by either truss-based discrete structural optimization or voxel-based continuum methods.

2.1.1 Discrete Structural Optimization

Discrete structural optimization refers to the parametric optimization of a discrete truss network composed of a network of nodes and linking struts. Discrete structural optimization is carried out by two methods: either geometric or topographic optimization (Christensen and Klarbring, 2008; Wang

2005). Geometric optimization refers to the optimization of nodal positions and associated strut size. Topographic optimization considers strut cross-section as the optimization variable, where cross-section can be set to zero, thereby altering the truss connectivity.

2.1.2 Continuum Structural Optimization

In this method, the microstructure of the available continuum is the optimization variable. Material distribution is varied in a binary fashion (i.e. voxels are defined as either solid or void) based on the continuum structural response to a specified a rejection criterion (Xie and Steven 1997). Where the properties of microstructures are varied as a function of their density (Bendsoe and Kikuchi 1988). The discrete nature of continuum optimization is compatible with CAE tools, but may introduce instability in the structure (Hassani and Hinton, 1999), and the resulting geometry is only conditionally optimal (i.e. identification of global optima is not guaranteed). Continuum structural optimization is categorized as either: Evolutionary Structural Optimization (ESO) or Bi-directional Evolutionary Structural Optimization (BESO).

2.1.2.1 Evolutionary Structural Optimization (ESO)

Evolutionary Structural Optimization (ESO) was developed by Xie and Steven (1993). In this method, CAE tools are applied to quantify the continuum response of the structure, which is initially defined as occupying the entire available spatial envelope. Voxels that do not satisfy a specified Rejection Criterion (RC), such as maximum stress or stiffness, are eliminated (i.e. set to void). By iteration of this process, an optimal geometry is revealed.

2.1.2.2 Bi-directional Evolutionary Structural Optimization (BESO)

In ESO, the voxel state cannot be reverted from void to solid, potentially resulting in a solution that is not globally optimal. An enhancement was proposed by (Yang 1999), in which the voxel state can change, i.e. material is removed or added as required. This bi-directional removal and addition approach results in improved convergence to the optimal geometry. However, an increase in the number of iterations to achieve convergence is cited as a disadvantage of BESO.
Whether discrete or continuum methods are utilized, methods reported in the literature are restricted to optimizing the objective function for a set of invariable constraints that are defined *a priori*. This limitation constrains the set of design problems that are feasible with TOP; in particular, the design of interacting components that are optimal at a system level is not possible with contemporary TOP methods. The novel method reported in this work allows boundary conditions to be defined as an optimization variable; thereby enabling a new class of design problems for topographic optimization.

3 OPTIMIZATION OF TOPOLOGY INCLUDING BOUNDARY CONDITIONS

Topographic optimization algorithms allow for structural optimization of a component subject to specified constraints and boundary conditions. In practical applications numerous individual components interact to achieve the required system objectives. For holistic optimization, it is therefore imperative that not only the individual component be optimised, but also the associated interaction; a strategy that is not possible with traditional topographic optimization algorithms.

3.1 Proposed topographic optimization algorithm

The topographic optimization algorithm of Figure 1 is proposed in this work to enable boundary conditions to be accommodated as optimization variables. The algorithm implementation is as follows:

1. ***Problem definition***

   Formal problem definition, including Initial Conditions (ICs), Boundary Conditions (BCs) and Rejection Criterion (RC).

2. ***Analysis***

   Algebraic or numerical modeling is completed as required to quantify the material response to the associated load scenario.

3. ***Modify geometry***

   Based on the material response and the associated rejection criterion, the voxel state is updated as required. For the example below (Section 4), if the voxel stress (σ) exceeds the allowable value σ_{all}, the algorithm will add material about the voxel in question according to a defined radius. In this algorithm the boundary conditions are modified *without conflicting with existing geometry* (Section 3.1.2).

4. ***Convergence criteria***

The optimization algorithm iterates until the convergence criterion is satisfied. The convergence criterion may include one or more of the following definitions: maximum number of elements, physical space available or the allowable number of iterations.

5. ***Report geometry***

Optimal geometry is reported, including the associated structural efficiency data.

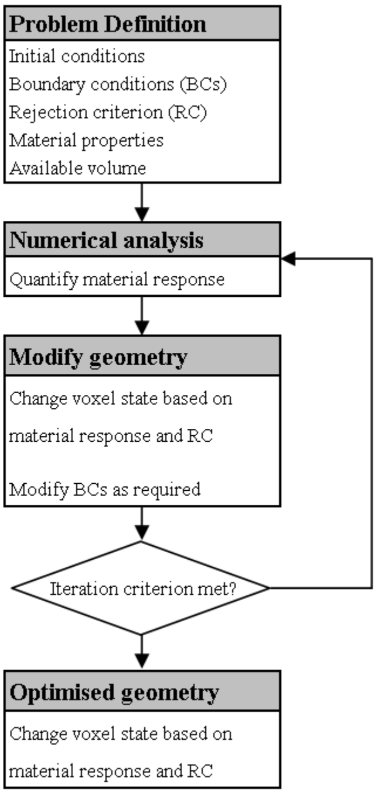

Figure 1. Proposed topographic optimization algorithm

3.2 Implementation hurdles

Enabling boundary conditions (for example component interaction) to be included as optimization variables provides significant opportunities for enhanced optimization outcomes, however a series of implementation hurdles exist. These are:

- Specific interaction is not known *a priori*
- Components cannot simultaneously share space
- Global optima may not be readily identifiable from the defined initial conditions

3.1.1 Component interaction

The optimal boundary conditions of interacting components are not known *a priori*. To allow the evolution of the boundary conditions of interaction of components, the Bi-Directional approach, as espoused by Xie and Stevens (1997) has been applied, whereby interacting components are defined in terms of the minimum material condition that allows the intended component interaction to exist. From this initial scenario, the associated boundary conditions of interaction are able to evolve to an optimal solution.

3.1.2 Material addition strategy

It is not possible for interacting components to share space. The strategy of material addition applied in this algorithm ensures that voxels can change state from void to solid *only if there is no conflict with* *the pre-existing geometry of interacting components*. Figure 1 defines an orthogonal voxel array with two interacting components that share the available continuum.

As indicated in Figure 1, the voxel at row 4, column 5 has been identified as meeting the rejection criterion. Consequently any neighboring voxels in the *void* state will change their state to solid (i.e. grey shaded). However, to ensure spatial constraints are met, the voxel at row 5, column 4 is forced to remain in the *void* state to avoid a spatial conflict of the interacting components.

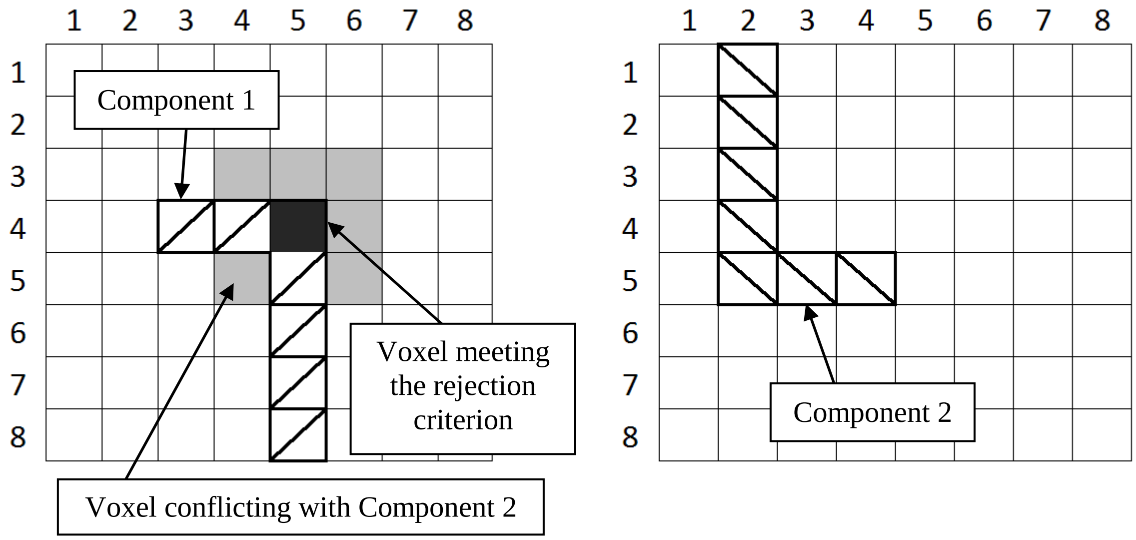

Figure 2. Material addition strategy.

3.1.2 Global optima

A particular concern within the topographic optimization literature is the conditional nature of identifying global optima, especially as the optimisation result is dependent on the algorithm applied and the associated initial conditions. When the boundary conditions associated with interacting components are defined as optimization variables, the difficulty in ensuring that the output is a global optima is more complex. Further work is required to ensure that global optima are identified in these scenarios.

4 APPLICATION

The proposed optimization algorithm enables a system approach to topographic optimization by allowing boundary conditions to be considered as optimization variables. The opportunities associated with this method are demonstrated by a simple case study of two interacting components that transfer a vertical tensile load via contact (Figure 3).

As discussed in Section 3.1.1, the components are initially defined by the minimum material condition that allows the intended component interaction to exist; this allows an initial CAE analysis without any spatial conflicts (Figure 3, Iteration 1). The subsequent CAE results identify that for this initially defined geometry, the allowable stress is exceeded. With CAE, the stress distribution is obtained for each voxel, and voxels with stress above the rejection criterion are identified. According to the stress distribution, the existing geometry is updated by adding material to the overstressed voxels conflicting with the pre-existing geometry of interacting components. Due to addition of material, the stresses are reduced (Figure 3, Iteration 2). Furthermore, the stiffness of the components is also increased, allowing a greater load to be transmitted before the components separate.

It should be noted that the component boundary, and associated regions of contact, evolves as the number of iterations increases, as seen in Figure 3, Iteration 3. The iterations conclude when the associated convergence criteria is met.

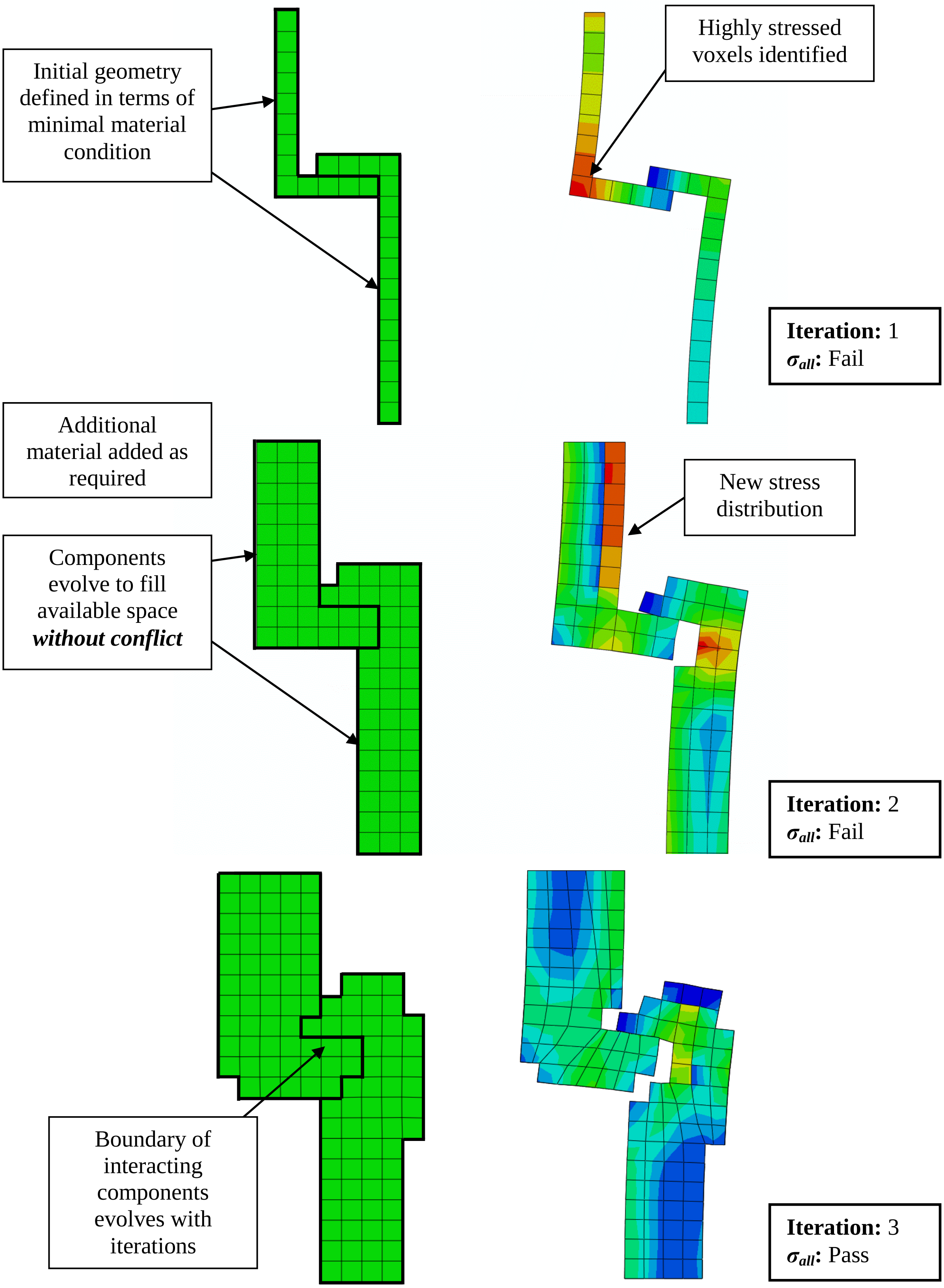

Figure 3. Proposed topographic algorithm applied to interacting components.

5 CONCLUSIONS

In this work, the authors have developed a novel topographic optimization algorithm that enables enhanced design outcomes by accommodating boundary conditions as optimization variables. This outcome extends the domain of valid problems to allow the systematic optimization of interacting components. This previously unavailable capability is of significance to holistic design as it allows the design of interacting components that are optimal at the systems level: a previously intractable design problem.

This method is demonstrated by the topographic optimization of simple interacting components. The developed algorithm was successfully applied to allow evolution of the interacting components based on specified initial conditions. The boundary conditions of interaction were shown to evolve with iterations of the algorithm to meet the allowable stress condition without spatial conflict between interacting components.

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