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IDEA MATRIX AND CREATIVITY OPERATORS

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ABSTRACT

Creativity begins with ideas. Scholars have addressed the conditions, factors, and processes to generate creative ideas. However, a formal technical framework to systematically manipulate ideas for exploration, evaluation and ranking against competing creative ideas remains a challenge. The present paper addresses this gap. We ground our work on scholars' definition of creativity. Next we formalize the notion of an idea as a construct of attributes and features using our matrix representation. We then propose a set of idea operators, which use matrix-expressed ideas as operands, to generate new ideas. We follow with structured matrix-algebraic methods used to assess, rank and measurably improve the new ideas in terms of creativity. To illustrate the application and utility of our methods, we assess, compare and improve two real-world competing product ideas. This approach, of using idea matrices and creativity operators to systematically generate and evaluate new ideas, presents a repeatable method to operationalize creativity.

Keywords: creativity, innovation, early design phases

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1 INTRODUCTION
Creativity is the ability to produce artifacts that are both novel and useful (Sternberg and Lubart, 1999; Simonton, 2004; Sternberg et al., 2002). This is the simultaneity property of creative work, i.e. the absence or either fails the creativity test. Exercising creativity is a process that begins with ideas in order to address a problem or an opportunity (e.g. Girotra et al., 2010; Twiesch and Ulrich, 2009). Without ideas, creativity is impossible. Our focus is on how to operationalize creativity for individuals, i.e.

- how to represent an idea using a consistent, but general, specification,
- how to use existing ideas to systematically generate new and potentially creative ideas,
- how to improve ideas in a repeatable way,
- how to rigorously evaluate and rank competing ideas.

We begin with scholars' definition of creativity. We then define an idea as a feature-attribute matrix. This enables the use of algebraic methods to operate on ideas. We propose a formal set of creativity operators that process our idea matrices as operands. Operations yield new ideas that can be evaluated and ranked using algebraic matrix methods. For ranking, we define our metric function that separates competing ideas by their creative intensity, which is determined by the distance relative to a reference creativity-matrix. The creativity-matrix is also used to improve ideas. Using our matrix analyses, we analyze the iPad and Kindle Fire tablet covers. We close with next steps in this work.

2 LITERATURE REVIEW

2.1 Creativity
Novelty and usefulness define the creativity of ideas and work products. Novelty means that an idea is original and surprising (e.g. Kaufman and Baer, 2004; Sternberg and Lubart, 1999; Simonton, 2000). Usefulness means an idea is endowed with utility and social significance. Temporal and social contexts establish the envelope in which usefulness and novelty are accepted (Simonton, 2006). But rejection does not mean an idea has no merit. Examples abound, Galileo was persecuted, Cantor was ridiculed, and Mendel ignored for 200 years. Creativity requires knowledge, skills, and motivation (Amabile, 1996). Serendipity exists, but is not reproducible. Intention and action are fundamental to creativity (Nickerson, 1999). Ability alone is sterile. Therefore, creativity is not the result of an event, but a process of sustained effort (e.g. Collins and Amabile, 2008). Creativity and process are inseparable (Plucker and Behetto, 2004). Many descriptions to produce creative work exist, e.g. "geneplore" (Finke et al., 1992), the eight step process (Hunter et al., 2006), the "creative cognitive approach" (Ward et al., 1999). However, scholars detect an actionability gap. To bridge it, Root-Bernstein and Root-Bernstein (1999) propose 13 thinking tools; Sternberg et al. (2005) present a taxonomy of creativity processes; Brown (2010) identifies 17 types of creative reasoning in the literature, and Spooner (2004) proposes a set of tools for interdisciplinary processes. Though their work informs creativity, operability remains a challenge. The operability of ideas is a focus of this article.

2.2 Idea manipulations
To operationalize process descriptions and principles, researchers and practitioners have developed heuristics and tactics to manipulate ideas. For example, ideas are embodied in words. Linguists use conceptual blending of words to create new meanings (Hampton, 1996). Emergence is recognized as key in this body of work (Estes and Ward, 2002). Emergence is a necessary condition for creative work. Brainstorming is a typical approach to foment the emergence of creative ideas from group interactions (Osborn, 1953; Sutton and Hargadon, 1996). Engineering has a rich repertoire of structured processes for individuals to exercise creativity. For example, Ulrich and Seering (1990) consider ideas as sets for engineering design and outline a process. TRIZ presents tools for resolving design conflicts (Altshuller, 2001). TRIZ is now being extended to other industry domains (e.g. Shouskov, 2007). But TRIZ principles require non-trivial interpretation and imagination. SIT is a method, which defines "creativity templates" and six operators. Janusian creativity is taught as five-step process (Rothenberg, 1996). Holland (1992) invented genetic-programming by combined principles from evolutionary biology and programming to algorithmically create new ideas.

2.3 Idea Evaluation
The resultant work's impact provides a way (e.g. Sarkar and Chakabarti, 2007) to evaluate an idea's creativity. Consensus is that the most creative work changes or creates a new knowledge domain

(Altshuller, 2001; Csikszentmihalyi, 1996,). This is called “Big-C” creativity (Beghetto and Kaufman, 2007) or “H-creativity” (Bowden, 1994). However, incremental improvements and adaptive works, known as “Little-c” creativity, are also creative (Kaufman, 2004). Altshuller (2001) posits five levels of creativity; from the incremental to the Big-C type. Beghetto and Kaufman (2007) describe their new concept of “mini-c” as “intrapersonal creativity that is part of the learning process”.

“A creative idea is one that expert groups consider creative” is a common practice to evaluate ideas (Amabile, 1996). For these tasks, frequently cited methods are: AHP’s pair-wise analysis (Saaty, 2000), utility theory (Keeney and Raiffa, 2003), and social science methods (e.g. Harsanyi, 1975). But scholars caution that individuals may apply social consequences and personal interests that bias a group’s evaluation (Janis, 1982; Hunter et al., 2006). Engineers use expert teams to evaluate competing ideas against a reference and to improve the most promising idea by “attacking the negatives” (Pugh, 1981). This suggests that ranking procedures must also include ways to improve worthy ideas.

The ability to evaluate and improve ideas is another salient element of this article.

3 MATRIX REPRESENTATION OF AN IDEA

3.1 Motivation and approach

Mathematics, where operators and operands are always rigorously defined, informs us on how to think about the representation of ideas and the specification of operators. As in mathematics, we define a set of operators using ideas as operands. Gärdenfors’ work on the geometry of thought and the representation of cognitive systems informs us on how to think about “a conceptual space built upon geometric structures based on a number of *quality dimensions*” (Gärdenfors, 2004). Ulrich and Seering’s (1990) *set based design* inspires us to think of ideas as operationalizable spaces. As in mathematics, except for trivial problems, a deliberate sequence of operations is required to find a solution.

We define an idea as a feature-attribute matrix and use matrix algebra to operate and analyze them. Inspired by Pugh (1991), we create a metric so that ideas can be found to be near or far from a constructed ideal reference-matrix. We enhance an idea by tuning its features to get closer to an “ideal” reference. Features define ideas (Devereux and Costello, 2004); they are an idea’s descriptors and specifications. They give identity to an idea. Features are independent variables and attributes are dependent variables. Attributes discriminate similar ideas. Attributes are values for usefulness and novelty that are determined socially as in Quality Function Deployment (QFD) (Cohen, 1995) and the Pugh method (Pugh, 1991). A compass’ salient feature is its magnetic needle that consistently points North. Anyone who needs to sail across an ocean will find this feature to be very useful. Its invention some 5,000 years ago makes the compass very novel.

3.2 Idea as a feature-attribute matrix

For an idea I^k, $f=(f^k_{1}, f^k_{2}, f^k_{3}, \ldots, f^k_{q})$ $q\ \varepsilon \mathbb{N}$ is its feature vector, and $\bar{a}=(a^k_{1}, a^k_{2}, a^k_{3}, \ldots, a^k_{p})$ $p\ \varepsilon \mathbb{N}$ is its attribute vector. The **novelty-matrix** N^k_{pq} for I^k is defined as a $p\times q$ matrix. The columns of N^k_{pq} are its *features* in domain specific terms. The rows are the *p attributes.* Each entry n^k_{ij} of the matrix has a value that associates attribute *i* with feature *j* for **novelty**. $n^k_{pq}\ \varepsilon$ [0,10] represents a ratio scale denoting the extent of a feature’s contribution to the novelty intensity (Saaty, 2000). The **usefulness-matrix** U^k_{pq} is similarly defined. In practice, the cell values are elicited from groups or surveys as in QFD (Cohen, 1995) and the Pugh method (Pugh, 1991). Given that creativity is defined as the *simultaneous* presence of novelty and usefulness, we define the **creativity-matrix** C^k_{pq} by the Hadamard product of U^k_{pq} and N^k_{pq}, i.e. $C^k_{pq}=U^k_{pq} \circ N^k_{pq}$. We eschew an additive approach because zero novelty or zero usefulness will additively indicate a positive creativity measure when, in fact, by the *simultaneity* definition there is no creativity. The multiplicative operation is therefore more appropriate.

3.3 Example of Apple’s iPad Smart Cover

Consider Apple’s iPad 3 Smart Cover (Apple, 2012) idea matrices (Table 1). Matrix cell values are subjectively generated by the authors (see section 5 comments). Table 2 explains the measures.

4 IDEA OPERATORS

Now we define matrix operations on idea matrices and illustrate their use. Space limitations constrain some sections to broad strokes. Section 5 provides a more complete example.

Table 1. Idea Matrices of iPad Smart Cover

	Novelty N^i_{85}					Usefulness U^i_{85}					Creativity $C^i_{85}=U^i_{85}\circ N^i_{85}$				
	front lid	magnetic hinge	magnetic closing	3-level fold	multi colors	front lid	magnetic hinge	magnetic closing	3-level fold	multi colors	front lid	magnetic hinge	magnetic closing	3-level fold	multi colors
Screen protection	7	2	0	2	0	9	0	1	2	0	63	0	0	4	0
Thin covers	3	0	0	3	0	9	1	0	3	0	27	0	0	9	0
Light weight covers	1	0	0	0	0	8	1	0	0	0	8	0	0	0	0
Easy installation	2	5	2	0	0	9	9	2	0	0	18	45	4	0	0
Hands-free reading	3	1	0	6	0	3	3	0	9	0	9	3	0	54	0
Hands-on reading	1	1	0	6	0	1	1	0	0	0	1	1	0	0	0
Sleep and wake up	0	0	7	3	0	0	0	9	4	0	0	0	63	12	0
Looks attractive	1	1	0	4	4	7	2	2	1	4	7	2	0	4	16

Table 2. Definition of intensity measures

Intensity	Definition	Explanation	Intensity	Definition	Explanation
0	none		7	strong	readily perceptible
1	weak	barely perceptible	9	very strong	high contrast perceptible
3	moderate	perceptible	10	dominant	governing
5	average	not weak nor strong	2,4,6,8	intermediate	values

4.1 Definition of an idea operator

Let $\mathcal{I}=\{ I^r \}$ be a set of ideas, $r\in\mathbb{N}$. An **idea operator** ω is mapping $\{\omega: \mathcal{I}\rightarrow \mathcal{I}$ and $\omega: \mathcal{I}\times\mathcal{I}\rightarrow \mathcal{I}\}$.
Let $\Omega=\{ \omega_i \}$ be a set of operators $i \in\mathbb{N}$, and $I^j\in \mathcal{I}$ and $\omega_i(I^j)\Rightarrow I^k$ where $I^k \notin\mathcal{I}$, i.e. I^k is a new idea $j\neq k$ where $\Rightarrow$ is used as a verb meaning "generates" or "transforms".
$\omega_i(I^j)\Rightarrow I^k$ where $I^k\notin\mathcal{I}$, does not assume I^j and I^k are necessarily located in the same domain. I^j can be in domain Δ, i.e.$I^{j\Delta}$, and I^k in domain Ξ, $I^{k\Xi}$, i.e. $\omega_i(I^{i\Delta})\Rightarrow I^{k\Xi}$. For simplicity, superscripts are not used.

4.2 Re-interpretation operator

Re-interpretation attaches new meaning to an existing idea by changing the mental models of the original idea (Plucker and Beghetto, 2004). For instance, Starbucks re-interpreted coffee shops as social spaces instead of take-out places. Analogy is also re-interpretive (Singh et al., 2009; Chan et al., 2011). It maps an idea to a new target domain. It selectively preserves mechanisms, structures, and causalities that remain meaningful in the new domain (Gentner and Markman, 1997). For example, the early design of airplane frames was largely analogous to the shape of birds. Restating an idea as a metaphor is also re-interpretive. A metaphor "interprets through … [a] comparison with something else", allowing the unfamiliar to be better understood and foster new ideas (Hey and Agogino, 2007).
We now re-interpret a greeting card business and a flower business. Table 3 (next page) shows the idea matrices. Features and attributes are self-explanatory. *Archivability* means that cards can be archived, but flowers cannot. The evaluation is from the authors (see section 5 comments).
This is an example on how to judge the creativity of an artifact using the norm of the creativity matrix and simple matrix operations. (An alternative way is shown in section 5). Norm is the *amplifying power* of a matrix (Strang, 1976), simple and intuitive. Norm of the creativity matrix for the card business is $\|\boldsymbol{C}^c_{53}\| = \|\boldsymbol{N}^c_{53}\circ \boldsymbol{U}^c_{53}\| = 232.8$, and for the flower business $\|\boldsymbol{C}^f_{53}\| = \|\boldsymbol{N}^f_{53}\circ \boldsymbol{U}^f_{53}\| = 226.6$. Both are equally creative. We note from $\boldsymbol{U}^c_{53} - \boldsymbol{U}^f_{53}$ that, relative to flowers, card's achivability is most significant. Although affordability and expressiveness are advantages, cards are weaker as a means of expression. From $\boldsymbol{N}^c_{53} - \boldsymbol{N}^f_{53}$, we note that, in terms of novelty, cards are weakest in aesthetics and expressiveness, but strongest in convenience and affordability. Entrepreneurs, noticing these facts, have created the "social expression" business where they offer other means of expression; such as toys, gifts, and other objects. Thus, American Greetings Corporation (2012) presents itself as "engaged in … social expression products". Cards and flowers are re-interpreted as "social expressions".

Table 3. Idea Matrices for greeting card business and flower business

	Card Business						Flower Business					
	Novelty N^{c}_{53}			Usefulness U^{c}_{53}			Novelty N^{f}_{53}			Usefulness U^{f}_{53}		
	Card	Message	Delivery	Card	Message	Delivery	Flowers	Message	Delivery	Flowers	Message	Delivery
Aesthetics	6	4	4	10	10	7	8	6	4	10	9	7
Convenience	10	8	7	10	8	7	8	8	7	10	9	7
Affordability	9	10	9	7	7	7	8	10	8	7	7	6
Expressiveness	8	8	5	9	9	9	9	8	6	10	10	8
Archivability	10	10	1	1	8	1	1	10	1	1	1	1

When using the re-interpretation operator, the operand matrix structure is expected to be used in a different context, i.e. $\omega_i\,(\boldsymbol{I}^{c}_{pq}{}^{K}) \Rightarrow \boldsymbol{I}^{s}_{pq}{}^{S}$. $\boldsymbol{I}^{c}_{pq}{}^{K}$ is the operand idea in the context of cards, *K*. Note that $\boldsymbol{I}^{s}_{pq}{}^{S}$ is created by preserving the feature-attributes of $\boldsymbol{I}^{c}_{pq}{}^{K}$, but remapping to domain *S*, social expressions. Attribute intensities of the resultant feature-attribute matrices needs to be re-assessed.

4.3 Abstraction operator

Abstracting is seeing the underlying simplicity of complexity (Wilson, 2003). Increasing a problem's ambiguity forces abstraction-driven creativity (Winger et al., 2010). One must suppress non-essential features so its structure, in its most frugal form, is revealed. Abstraction reduces cognitive load to facilitate fresh thinking (Spooner, 2004; Davidson, 2003). Research shows that more creative products are likely from abstract characterizations of a problem than otherwise (Condoor et al., 1993). Abstraction must necessarily start from something that exists. For example, Amazon Books is an abstraction of a retail store, but with intense retail-like attributes. Eliminating too much detail is risky, and omitting valuable information is deadly, e.g. Apple Maps on iOS6. Effective abstraction is a delicate balance.

To abstract an idea in its matrix representation, one can use matrix algebra to identify the important features, their resultant attributes and suppress the less important features. (Section 5.2 provides an algebraic method to rank features and attributes by their relative importance), or reflect on whether a different idea using only the important features will yield a new or a better idea.

For instance we now consider the Kindle Fire Lightweight MicroShell Folio (Amazon, 2012). Matrix entries in Table 4 are from the authors. Table 5 shows the relative weights of importance of the folio's key features and attributes, given by the Perron eigenvectors of feature and attribute correlation matrices (see section 5.2 for the justification of this ranking method). 2-level folding, elastic strap, and multiple colors are most important to overall creativity. Only hands-free reading, looks attractive, and screen protection are relevant to creativity. This product impresses users as a book stand. Thus, the abstraction of the Kindle cover is simply a book stand.

4.4 Pattern Creation operator

Pattern recognition and creation are cognitive functions critical to creativity (Pi et al., 2007). Patterns are structured connections based on some working principle of the elements in a set. Principles are the basis of coherent patterns. Ice crystals' hexagonal shape is a pattern; the principle is geometry. Physics is about patterns of nature; principles are the laws of nature. Patterns order and structure the world.

Consider Table 3. The U^{c}_{pq} matrix reveals that for flowers and cards aesthetics, convenience, and expressiveness are the dominant attributes. In both cases, the artifact is the dominant feature. These observations define a pattern for these two businesses. An entrepreneur can properly infer that a different artifact can have the same attributes and create the new business domain of "social expressions". Absence of a pattern can also lead to dramatic insights. Cantor's diagonalization proved the absence of a pattern and conclusively established the non-denumerability of irrational numbers.

The first step in pattern creation is the extraction of key features or attributes from an idea. Second is finding governing structural or dynamic relations, from the features or attributes alone or together. One can use tables, flow diagrams, entity structures, or other tools to search or form thematic regularities, archetypes, or the like for patterns, and test if patterns appear or can be formed.

Table 4. Idea Matrices for Kindle Fire Lightweight MicroShell Folio

	Novelty Matrix N^k_{85}					Usefulness Matrix U^k_{85}					Creativity Matrix C^k_{85}				
	Front lid	Polycarbon back	Elastic strap	2-level fold	Many colors	Front lid	Polycarbon back	Elastic strap	2-level fold	Many colors	Front lid	Polycarbon back	Elastic strap	2-level fold	Many colors
Screen protection	0	0	1	0	0	5	1	1	2	0	0	0	1	0	0
Thin covers	2	1	0	0	0	2	0	0	0	0	4	0	0	0	0
Light weight covers	0	2	0	0	0	2	3	0	0	0	0	6	0	0	0
Easy installation	0	0	0	0	0	1	1	1	0	0	0	0	1	0	0
Hands-free reading	0	0	1	3	0	2	2	2	3	0	0	0	2	9	0
Hands-on reading	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0
Sleep and wake up	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Looks attractive	0	0	0	3	2	0	0	0	1	2	0	0	0	3	4

Table 5. Relative priority of features and attributes of the Kindle Fire Lightweight Folio

Priority	Features	Perron eigenvector of $\boldsymbol{C}^{k\ \mathrm{T}}_{85}\boldsymbol{C}^k_{85}$	Priority	Attributes	Perron eigenvector of $\boldsymbol{C}^k_{85}\boldsymbol{C}^{k\ \mathrm{T}}_{85}$
1	2-level folding	0.970	1	Hands-free reading	0.934
2	Elastic strap	0.193	2	Looks attractive	0.358
3	Multiple colors	0.147	3	Screen protection	0.020
4	Front lid	0	4	Light weight covers	0
5	Polycarbonate Back	0	5	Sleep and wake-up	0
			6	Thin covers	0
			7	Hands on reading	0
			8	Easy installation	0

4.5 Synthesis operator

Research supports the claim that combining existing ideas can lead to new ideas (e.g. Devereux and Costello, 2004; Costello, 2000). The ubiquity of the plus sign, +, and the integral, ∫, in journals is thus not surprising. Synthesis is a special kind of combination; it produces emergence, i.e. synergy. For example, *frenemy*, by blending two words, creates a new one, very rich in semantic meaning. Emphatically, Maxwell's four equations is a Big-C synthesis. Synthesis can result in unprecedented creative ideas, e.g. the iPad™ is the synthesis of camera, photo album, music player, GPS, computer, and apps. Synthesis of Darwinian evolution and programming created the field of genetic programming (Holland, 1975). Boeing combined the fuselage and wing into a single blended-wing airplane unit. This design increases the surface area to provide more lift and lower operating costs (Jayanth, 2012).

We now sketch how the synthesis operator can be used algebraically. Consider two ideas, I^1_{nm} with feature vector f^1_n and attribute vector a^1_m, and I^2_{pq} with f^2_p and a^2_q vectors. The synthesis of the two ideas is represented by idea-matrix $S^s_{n+p,m+q}$ with feature vector $f^1_n \cup f^2_p$ and attribute vector $a^1_m \cup a^2_q$. The entries in this matrix are revaluated using domain knowledge of the target domain. To evaluate whether this combination is synergistic, we test whether $\|S^s_{n+p,m+q}\| > max\{\|I^1_{nm}\|, \|I^2_{pq}\|\}$. If so, this suggests synergy that produced improved attributional results.

4.6 Fractionation operator

Fractionation is based on the principle of "divide and conquer". It decomposes an idea into its parts, then selectively using them with/without other ideas to create new ideas. Fractionation is not random. It must be grounded on the structure and principles of the system. Principles may be apparent or obscured. For example, engineering fractionation can be based on function, material, energy flows, or modularity (Otto and Wood, 1991; Hölttä-Otto et al., 2003). Business model fractionation can be based on functional structure, such as outsourcing. Industry sectors can be fractionated based on physical and non-physical dichotomy, such as services (Tang and Zhou, 2007). Knowledge domains can be fractionated based on laws of physics, as in Newtonian and quantum mechanics. Gamper is a leading artistic fractionist (Musical Chairs, 2012). From his vast collection of used furniture, he disassembles them and remixes the pieces into furniture of surprisingly useful and clever configurations. The organizing principle is usability.

4.7 Reversal operator

Reversal is an example of divergent thinking. Reversing the logic structure of an existing idea often leads to new ones (Rothenberg, 1996; Esters and Ward, 2002). Open source software development reverses the in-house development practice. FedEx reverses the axiom: "the shortest distance between two points is a straight line". Whatever a package's destination address, FedEx ships them all to Memphis where they are sorted and immediately dispatched. This is true even for overnight mail delivery sent from Manhattan to Brooklyn. A demanding form of reversal is Janusian thinking - the original problem and the opposite ideas must co-exist simultaneously (Rothenberg, 1996).
Engineers used reversal to design the Active Noise Cancellation system (Active Noise Cancellation, 2012). It cancels unwanted sound by generating *more* noise of equal amplitude but of *opposite* phase. British engineers solved the car engine vibration problem similarly. In their solution, half the pistons move in one direction and the other half in the *opposite* direction to cancel vibrations. And for vibrations generated by the longitudinally different displacement of the pistons when moving in opposite directions, new balancer-shafts rotate in the *opposite* direction of the rotating pistons.

5 EVALUATION OF CREATIVE IDEAS

5.1 Idea comparison

We draw inspiration from the scholarship of two scholars, Pugh method (1991) and Gärdenfors (2004) whose study of the representation and analysis of ideas and concepts in a geometric space. Our goal is to rank competing ideas in terms of creativity and to improve the most creative idea. Given ideas $1,2,\ldots,n$ represented by their matrices $I^1_{pq},\ldots, I^n_{pq}$ $p,q \in \mathbb{N}$. We form the *reference matrix* $B_{pq}^{\star}$, each entry defined by $b_{pq}^{\star} = max\{i^1_{pq}, i^2_{pq},\ldots, i^n_{pq}\}$. The distance from an idea to the reference idea is calculated using the *Frobrenius* norm of the distance matrix, written as $D^k_{pq} = I^k_{pq} - B_{pq}^{\star}$, $k=1,2,\ldots,n$.
Competing ideas are ordered by their distance to the reference idea, according to novelty, usefulness or creativity. The best idea in the comparative group has the $min\{\|D^1_{pq}\|, \ldots ,\|D^n_{pq}\|\}$.

We define the distance, $\rho(\alpha,k)=\| I^{\alpha}_{pq} - \boldsymbol{B}^k_{pq}{}^{\star}\|$ as the **creative intensity** of idea α relative to the reference idea. (I^k_{pq},ρ) forms a metric space and if $\rho(\alpha,k)>\rho(\beta,k)$, then I^{β} is more creative than I^{α}. *N*ovelty and usefulness intensities are similarly defined, depending on the type of matrix analyzed.

5.2 Idea improvement

The top-ranked idea can be further improved by "attacking the negatives", i.e. selectively strengthening features or attributes by reverse engineering the later. Define $A^k_{pp} = I^k_{pq} I^{k\,\mathrm{T}}_{pq}$ as the **attribute correlation matrix** and the matrix $F^k_{qq} = I^{k\,\mathrm{T}}_{pq} I^k_{pq}$ as the **feature correlation matrix.** Both A^k_{pp} and F^k_{pp} are symmetric.
Assuming that attribute i's correlations to attributes which are important lend attribute i more importance than its correlations to less important attributes, one can model the importance weight of attribute x_{i} to be proportional to the sum of the weights of all attributes that are correlated to i, $x_i = \lambda^{-1} \sum_j A_{ij}\, x_{ij}$, or in matrix form $\lambda\mathbf{x} = \mathbf{A}\mathbf{x}$, where λ is a scaling constant and vector $\mathbf{x}$ is the vector whose elements are the x_i, i.e. the importance weights of all attributes. The importance weights are the elements of an eigenvector of **A**. The Perron–Frobenius theorem tells that only the Perron eigenvector, i.e. eigenvector corresponding to the largest eigenvalue, has all elements non-negative.
Thus, one can use the elements of the Perron eigenvector of A^k_{pp} as the relative importance weights of respective attributes. The same applies for the features of F^k_{qq}. With such weighting, engineers may choose to focus on the most important features and/or improve the weakest ones.

5.3 Example of two competing product ideas

We compare the creativity of the Apple iPad 3 Smart Cover and Kindle Fire Lightweight MicroShell Folio. Versions of the products were as of May 2012. Their idea matrices are presented in Table 6.

Table 6. Evaluation of two competing ideas

Evaluations		iPad Smart Cover		Kindle Fire Folio	
Novelty Intensity	$\rho(N^i_{pq})$	3.317		13.675	
Usefulness Intensity	$\rho(U^i_{pq})$	3		20.928	
Creativity intensity	$\rho(C^i_{pq})$	6.403		115.98	
Attributes ranking by $\boldsymbol{C}^i_{pq}\boldsymbol{C}^{i\ \mathrm{T}}_{pq}$ Perron Eigenvector		0.733	Screen protection	0.934	Hands-free reading
		0.386	Hands-free reading	0.358	Looks attractive
		0.351	Thin covers	0.020	Screen protection
		0.342	Easy installation	0	Light weight covers
		0.231	Sleep and wake-up	0	Sleep and wake-up
		0.110	Looks attractive	0	Thin covers
		0.090	Light weight covers	0	Hands on reading
		0.014	Hands on reading	0	Easy installation
Features ranking by $\boldsymbol{C}^{i\ \mathrm{T}}_{pq}\boldsymbol{C}^i_{pq}$ Perron Eigenvector		0.869	Front lid	0.970	2-level folding
		0.392	3-level folding	0.193	Elastic strap
		0.218	Magnetic hinge	0.147	Multiple colors
		0.207	Magnetic closing	0	Front lid
		0.023	Multiple colors	0	Polycarbon back deck

The iPad Smart Cover has higher creativity, novelty and usefulness intensities than the Kindle Fire Folio (Table 6). From a set of newly generated ideas, designers will be naturally interested in improving the best idea by looking into its features and attributes. For the iPad cover, the most important attributes are: screen protection, hands-free reading, and thin covers, and the most important features are: front lid, 3-level folding, and magnetic hinge. Data on the relative importance of individual features and attributes can guide designers' choices and strategies to improve an idea.

6 CLOSING REMARKS

Our goal has been to make a contribution in the study of operational creativity. We focused on the representation, operations, and evaluation of ideas. The idea matrix and operators together show the potential for a coherent system of creative operations of ideas (i.e. ideation) that individuals can conduct with repeatability. We performed the evaluations in this paper; a more rigorous approach to verify our proposed technical framework is necessary. Our action plans are to: (i) test our idea matrix and operators with challenging problems, (ii) test for internal and external validity, (iii) investigate the extent to which our idea matrix and operators span the problem space, and the extent of the operators' orthogonality, and (iv) derive a set of normative principles to buttress the rigor of our approach.

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