

GEOMETRY, XOROS, SYNTHETIC PRINCIPLES

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Abstract: What is the role of mathematical thinking when addressing logic address and paradox? An attempt to explore the educational significance of a relationship, with a variety of instructional implications.

Pythagoreans developed a mathematical aesthetic vision of the universe, believing that everything in nature is harmoniously connected to a system of numerical ratios. Convinced that everything in the world ratios is natural numbers they reached number Phi, the golden section. The ancient Greeks called it "divine proportion" as it appears in "Elements of Euclid." The problem of Plato, timeless philosophical problems and crucial in today's world: the imitation, in art and architecture. The question of the extent of its use in educational practice as to influence public opinion is modern, but originally came from the Platonic problem of 'imitation' -artistic imitation, as a teaching instrument, to approach true knowledge. A search for points, where art-architecture and mathematics meet and interact.

Keywords: *Mathematical thinking, art, architecture, aesthetics, educational significance*

1. Introduction

"Greek Geometry consisted the uncorrupted model, proposed not only for each knowledge aimed at perfection, but also an unparalleled prototype of typical characteristics of European thought ... These columns, capitals, architraves, entablature, drooping ornaments without ever leaving their position and mount them, make me think of those joints of pure science, as the Greeks had captured, definitions, axioms, proposals, theorems, conclusions, findings, problems ... namely the visible machinery of the Spirit." (Paul Valery)

"Let no one with no knowledge of geometry, be accepted", Plato describes in his dialogue "Timaeus" the world, as a geometric composition of body harmony, the famous 5 Platonic Solids, that will influence the Renaissance and will cause many of the time enlightened spirits be troubled and intrigued.

"When I talk about the beauty of shapes, it by no means insinuates what the common perception would count as beauty in a living being or on a painting; I describe a straight line, a circle, two dimensional surfaces and solid shapes, that consist of lines like this, as primary elements. . . as they are not pretty when compared with others, but enclose the natural beauty in the eternal essence ...»(Plato, Filivos dialogues)

An attempt to deepen to the least, the least we see, not hear or smell, are through art and mathematics. Mathematics with the language of logic. Art by intuition, emotion. Then what is justice, freedom? Here we are talking about the world of ideas and concepts, as Plato defines.

In the common sense, the connection of mathematics, geometry and the composition of space are considered as indiscriminate directions with diversity in thinking and the scope of their implementation. Gradually the perception of space as a geometric solid is evolving (Zafiropoulos), although undocumented, with rules of conceivable perspective, through experimenting the application of geometric relationships and introducing a sense of proportion and harmony.

2. Mathematics and art connection

2.1. Philosophical background

Is there a relationship between Mathematics and Art? Science and art in general? Mathematics certainly has an element of objectivity and is related to the logical part of man, as opposed to art that has an element of subjectivity and is mainly related to the emotional part. So in principle, there seems to be a great unbridgeable difference.

The theme of art and its relationship to the truths of mathematics and science as well as many other courses has been reviewed extensively in the Platonic and Aristotelian work and the *Likes* in the classical period.

Plato with over 100 references in art, particularly dealing with it, places her under fierce criticism (with the exception of architecture) considering it a mere imitation of imitation, that is imagery and copy of the sensible world, which in turn is a copy and illustration of the invisible world of ideas. So he thinks that is three times further from the truth. The Art (as producing empirical copies and conjectures) and Mathematics, as the means of search for the noble and blameless, are evaluated and placed by Plato in specific locations in the famous line of dialogue, called "State-Politia". One can agree or disagree with the Platonic positions about the nature of the truth of mathematics proposals compared to the nature of truth or not of the creations of art. Yet, surely one can not ignore the debate introduced by Plato, because of the problem of "imitations" of art is investigated by the great philosopher in general for all the "imitations." And imitations when interpreted as representations of concepts consist an essential tool of science in general, but in particular in the teaching of natural sciences and art.

Various theories were based on the linear motion and contact, that is developed between the environment and the organs of vision and hearing of people. Plato argues that rays are arising from the eyes and gradually lose their intensity as they travel on the environment to meet the objects. Aristotle admits that the vision and hearing are based upon the transfer of the air waves in the air, and can only be explained in detail through correlation of spatial and temporal divisions. In his works *Physics* and *about Soul*, he is working on a linear analysis in both time and space (Zafiropoulos S).

The Aristotelian view to art, is clearly more advantageous than that of Plato, having dedicated a whole book (the "Poetics") where the definition of "tragedy" comes from "...so tragedy is mimesis of great and spotless act ... catharsis." The tragedy-which is art, leads to the clearance and not the deterioration of the soul, as Plato claims.

Einstein says on the subject: "Where the world ceases to be the stage for personal hopes and wishes, where we as free beings, observe in amazement, wonder about and study it, there the realm of art and science begins. If you translate what we felt and saw through the language of logic, then we do science; if we show it through forms, inaccessible to conscious thought, but identified with intuition as mergers of meanings, then you do art. The common denominator in the art and science is the devotion to something that goes beyond the personal that lies beyond the region of arbitrariness. « (Eco, 2004).

Our view is that any artistic creation in the visual arts, once shown on a canvas or space contains, no doubt, conscious or unconscious use of geometry and proportions in obvious or less obvious form. Many have sensed that art has some secret, sacred meaning, at times. If we accept that the geometry

existed as an innate instinct in human nature, it had long been before it became conscious logical sense. The discovery of geometry is therefore an important triumph of man and the elements themselves. As if the "door" of capabilities opened. Practically "observing" is something completely different than planning. The intellectual capacity of design is dictated by the laws of geometry. Thus geometry developed in the necessities of life, was stabilized of practical rules that have entered from the experience and implementation, measuring and comparing.

As link elements of the natural and built environment together with humans, conceivable axes lines are invented, that gradually evolve into an essential tool of perceptual organization of the environment. Their use allows people to perceive the space as a systematized solid.

Euclid studies and develops the theory of perspectives, which establishes the importance of the *measure, measurement of the geometric relationships* with *harmony*, in every level of human scale. In his book *Optics* he studies the effects of various geometric space correlations, and the process of harmonization, of old tradition-axis lines of the built and natural environment.

Important part of modern art, with pioneers the Bauhaus experts etc. consciously uses geometric shapes in the creations. An art form e.g. a statue is a tangible intervention in three dimensional space, which is not always a mix of strict geometric solids, its various parts, consciously or unconsciously incorporate an analogy, one that befits the artist's mental targeting. These ratios had a particular target, e.g. in the classical era, focused on the depiction of the ideal citizen - a fighter for the noble and spherical perfection and certainly does not require good knowledge of mathematics and geometry, but the outburst of artistic talent (Michelis, 1940).

2.2. Historical background

"Geometry has two great treasures. Pythagorean theorem and the golden section ... »Johannes Kepler. The golden number Phi (the mathematician Marc Barr, in the early 20th century. honorably proposed to use the first letter of the Greek sculptor Phidias), is widely used in the space design, industrial design, fine art. Western thinkers with controversy interests were fascinated by it, to at least 2400 years. The Greeks believed that mathematics originated in Egypt from which they "borrowed" their empirical theories, which later, developed and documented. According to Mario Livio, mathematician, astrophysicist, author (1945 -) some of the greatest mathematical minds of all times (Pythagoras, Euclid, Leonardo of Pisa, Johannes Kepler, Roger Penrose), spent endless hours in the study of the golden section and its characteristics, thus it became the largest all-time inspiration in the history of mathematics.

The ancient Greek mathematicians studied its scientific properties due to the frequent presence at the natural elements and geometry. Its inspiration and in depth evolution is attributed to Pythagoras (560-480p.Ch), who proved that it formed the basis of the human body proportions. Plato (427-347p.Ch.) based on mathematical principles of Pythagoras's mathematical principles, inspired the *geometric progressions*. His theories of *beauty* deeply influenced the theoretical thinking of future generations, focusing on the principles of *measure, symmetry, unity*. Euclid (325 - 265p.ch.) first documented and recorded the definition of the *golden section* in his book *Euclid's Elements* (300p.ch.). The first who had set a sense of the golden section proportions, and the properties of the rectangular triangle with aspect ratios 3/4/5 (holy triangle), were the Egyptians (Great Pyramid of Giza, Cheops, tactile representations).

The wide application of the properties of the golden section and harmonic alignments are found a) in the sculptures of ancient Greece, b) temples) in Greek theaters (Epidaurus, Dionysus). These ratios were the "measure" for determining the morphology and aesthetics of the built interiors. The man as a composer and creator, invented rules of relationships between forms of composition, i.e. a) the invention of *emvati*: segment of geometric measure and analysis, which standardized the proportions of the building outside and inside, while focusing on aesthetic upgrade, b) and then the *grid*, the standard frame where *emvatis* will be integrated to synthesize the unity.

In ancient Greece, parts of the human body were considered as units for *emvatis*. The *grid* was simply a means of preserving the basic proportions, having a minimum dimension *emvati* as a basis for which all other ratios were multiples of. All aimed to aesthetic perfection. The "visual distortions" promoted

- through a strict grid system - freedom of composition, the adaptation to environmental conditions (e.g. sunlight intensity), and the correct visual perspective.

In Egypt, however, the "rules" of standard sizes corresponding to each individual part of the human body did not allow a "visual distortion", ignoring perimeter body ratios, motion performance, and neutrality. So the necessary optical transformations are excluded.

Later, Vitruvius (app. 80-70a.C. – 10a.C.), in his work *About Architecture*, refers to *analogies* which are more metric systems that determine the size of the project, so the goal is not an achievement of beauty, but its realization through the proportions of its parts. During Renaissance the principles of *perspective display* were developed (Leon Batista Alberti 1404-147b.C., Leonardo Da Vinci 1452-1519b.C.), which were primarily quoted in the scientific works of Euclid, Thales of Miletus.

The years that followed, technological developments removed the mental processing of space composition, off the harmonic geometric proportions. The researchers focused on the technological knowledge of materials and technical properties offered, *the grid* was just the basis for a most correctly assembly of identical elements, aiming at impression, economy, "virtual reality". The Swedish architect Le Corbusier (1887-1965) focused his entire design philosophy on golden ratio-scale systems – the Modulor (Corbusier, 2005) -, yet, not for the sake of harmony, but for mere beauty reasons.

3. Design methodology issues

3.1. Early stage, psychology

The idea, the conception, the pre-planning, are cognitive processes, where, social affairs, location, moral values, human-art and technological developments relationship, are “players” involved in a "creative language" for the synthesis of interior architectural systems. These systems as structured sets, are seeking *harmony* as the basis of creation. (Zahariadis, 2008)). The relationship is bidirectional, however, the *work* will be in maintaining dialogue with the image of diverse human environment, requiring the concurrence of creation-landscape, guided by *stimuli, sensations, imagination* - the human perceptual ability to recruit-absorb to create something new. This implies an *active mind*, cognitive processing abilities beyond mere imitation, so as to recognize, anticipate, and meet the needs of the human body, to the extent that the result will fulfill the “ideal of beauty”.

The concept of *beauty* attracted many philosophers, thinkers and therefore designers-artists in the past. Let's not forget that an artist of spaces was required to have comprehensive knowledge of philosophy, astronomy, mathematics, arts. His involvement with the built space presupposed not only to meet the functional and physical needs, but the spiritual quests of the era, that would be applicable in the *alive space*. Pythagoras tried to interpret the universe, to understand the meaning of musical proportion and the concepts of *metro* and *harmony* through the ratio of the numbers. Plato, later, interpreted *beauty* as an unending, preexisting, perpetual value, of beings, places and time. It belongs to the world of ideas, incorporate the *measure, symmetry, unity*; it is imperishable, perfect, the highest authority, the backbone of ethics and aesthetics of things.

The human eye is a sensitive tool, which may directly, even without practicing, recognize a line that is not perfectly straight, vertical or horizontal, or a curve not continuous and normal in its curvature. Yet, it is not so sensitive in understanding when the angles are not correct or the spatiality of forms, at least when they are not clearly broken down into forms composed of basic geometric shapes (Lianos, 2007).

The examples are many. The ability to immediately recognize the form is indispensable condition for capturing the architectural message. The more typical and less confusing, the simpler and more normal the forms are the more easily understood and recognizable they are.

3.2. Primary steps

Geometry is the science of the properties and relationships among figures in space. It is therefore directly linked to the quality and properties of forms in interior spaces and it allows us to create two-dimensional images, able to build and control three-dimensional forms in space.

During each synthetic processing, the ultimate goal is to build those spaces that express the purpose of the job. For this reason, we often have, while processing complex draught-designs necessary for the achievement of the final goal (building, spaces), to verify at any time the "geometric medium "(plan) with the implementation of the object, in order, the "Geometric sake" not to replace the synthetic process. We should be very careful not to become "servants" of the Charm of geometry, which is different from the principles of synthesis.

As known from the analysis of a building work, exteriors and interiors, we define the general theoretical form of the building, study the available interior space, the surroundings, the precise accommodated functions, and then move -scribble- on to the composition of the building spaces (Sfaellos, 1999).

To control our ideas, we need a design promotional tool, geometry, with which we cut, set and create the space. But the three-dimensional forms in architecture is not the outer of a volume, e.g. as occurs with a sculpture, but the surface enclosing a space, as space in architecture is not empty, but it is the volumetric space in which a coexisting range of activities occur. To extend this, in the case of Xoros design, "inspiration - creation" refers to an organized system space, where we experiment through its use, and perceive through its form.

When it comes to designing interiors, there is an inherent and inseparable relationship between design and proportion. For some this comes naturally, to others, not so much. One of the reasons why some people seem to be "inherently" inspired designers is their ability to use geometry with a flattering way in any space. The space of shapes is the set of lines, objects, subject to certain rules and restrictions, where man is in a perpetual motion "in response to feelings" to the dominating forces and contradictions (Mihelis, 1940). All the elements that compose the interior space, are carriers of meanings, feelings, experiences, is the result of the externalization of the designer's intent to move from the imaginary space to the project implementation, *the volume under the light*, as assisted by the geometric shapes and volumes. The plasticity of this internal space depends on the ability to convert the insight into "*objects of visual, auditoria, kinesthetic stimuli*".

When referring to the synthetic process of the interiors, we refer to deliberate choices of elements whose properties are characterized by the geometry: *the scale and size, the ratio - the relationship of the parties forming the space -, the theatricality (harmony or contrast, unity, variety of contrasts, rhythm)*. The starting point is the "inspiration", the central idea (Fatouros 1979).

3.3. Best alternative methodology

There were attempts at experimental analysis methodologies in interior spaces. for creativity and originality in the result. The existence of objects around us, creates the sense of space, utilizes it in combination with systems and organization. Analyzing this organization, we define the relations between the "central idea" and the final outcome, i.e. the *signifier and signified*, the concept and materials.

3.3.1. Prerequisites

Three types of analysis were explored and implemented: a) the frequency of use of space zones, b) the equivalences of places and functions. Thus, the classes of interior spaces are differentiated, depending on the context that determines their function, c) a detailed general criticism, as a generalized approach of the previous two.

The format that will be formed is determined by three essential "duties": a) provides the aesthetic appearance of the project; b) is information "message", carrying messages and expressing the contents and properties of the project, c) evolves into a symbol.

What follows is an evaluation and critique of the data, so as to create a) places with a sense of security, b) private and public interior spaces.

Considering that the body is the main frame of reference, we find that the structure of forms, results in the projection and visibility of the human form in its natural environment.

The design *intervention* to adapt the environment to the intended purpose, uses, naturally, these morphological structures, which depend on: a) the creative concept, b) the structure of organized forms, c) the conceptual evolutionary relationship of the *signifier - signified*.

We feel the need to justify, even retrospectively, our selection of forms that meet the efforts for environmental organization. Their contribution to a qualitative analysis, which supposedly, can be assessed objectively and scientifically, is allowing us to verify, confirm and correct the aesthetic quality of our judgment. This verification takes place for, rather a psychological and moral assurance and encouragement (Emmer, 2005).

3.3.2. *The Technique*

The computational mathematical technique helps the methodology to achieve the purpose of creation. It sets rules for the organization, makes our spatial interventions effective.

The design that sets the framework of human activities has a dual dependence on technique (which allows us to make visible our imaginative creations, as a means to organize the material field, for the uptake of any performances):

- A) Immediately: perfecting the manufacturing capabilities,
- B) Indirect: the landscape ceases to be natural, and progresses into a technical one.

The design raises questions of form. The technique offers solutions, offering an original source of inspiration, according to the aesthetics of the time.

For the composition of the forms, we adopt two strands. The first analyzes the relationship among internal space forces, and decomposes them into their first elements, depicting them through alignments. The analysis progresses from a general to a specific level. The second goes backwards, since it is used for the synthesis of form. It reverses the chain of reconstructing the data analysis of the first part.

The character in the Greek conception of space is the free three dimensional developments in the area, so as to occur under the most advantageous angle to the viewer. The seemingly free, but actually coordinated with the movement in the environment, alignment consists the means for successful integration with the human factor.

By design, we attempt for a coexistence of the spiritual purpose and the pursuit of aesthetic satisfaction, so that *visual inspection of the design quality, and speed of use* is achieved, as indicated by the fulfillments of aesthetics needs. The design is constantly in search, to find parallel aesthetic and functional forms (Calter, 2008).

3.3.3. *Main Stages of Planning*

Through the above parameters, we enter the main production stage for the project, and interior design process:

1. *Synthetic and Functional Organization.*

It includes: a) research, recognition, understanding, evaluation of structural and morphological characteristics, b) analysis of research findings, c) composition, functional and aesthetic intervention.

2. *The New Synthesis, namely:*

- i) *From the source of inspiration, ii) to the central idea,*

Whatever inspires us is the core of the study (in the natural or human environment, philosophical Ideas, historical data of the past), anything of minor or significant value in life.

iii) *In the composition.*

Given the element of inspiration, thoughts and dreams take shape, are articulated in spatial forms, narrate the relationship space - user. The *forms* will be the *vehicle* of any aesthetic feasibility of the technical capabilities, the ideological beliefs, and the economic situation of the user. The *hidden geometry* will provide the organic structure of *forms*, through *decomposition, simplification, interpretation* of the characteristics of the central idea.

To come up with creative results, we follow some experimental, but effective routes.

A. Market research (as analyzed before)

B. Decoding of the central idea. Thoughts, feelings, semantic and morphological processing of the elements of inspiration,

C. Recording of the above, on expressive focused processing tables (emotion-forms-shapes-material-colour), under the distinct eye of the creator, grounded on theoretical basis and methods of expression.

"The goal is to bring communication to the spirit, heart and soul of the user, to help him feel the proposed designed space" (Haralabidou, 2012).

D. Alignments, i.e. scribbling on paper with comprehensible and linear lines, resulting from the recording of the characteristics of inspiration, but also by the geometry of the interior space. From this stage, the rhythm, form, shapes, will emerge, based on the organization charts. It is the main stage of creation, since now, the ability, creativity and inventiveness of its creator, is discerned.

E. Organization charts, which are divided into:

a) *the plain one*, i.e. we process i) the articulation of the relationship of the functional modules, based on our knowledge of anthropometry, ii) we move to deepen into the way of their development, based on the concept of the volume, the mass, and the movement of the user,

b) *The synthetic chart*, with a composition of geometric shapes. Based on the sequence of the produced geometric shapes, the designer "judges upon, remodels, and selects the aesthetics, the hierarchical and unifying relationship between the activities of the site, criticizes, experiments, through the alternative operating proposals - materials - colours, giving birth to the final two-dimensional and three dimensional effect.

F. Review of the designer's endorsement, made through a) his perspective view, b) the user's requirements, c) the use and the particularities of the space.

"The plasticity of any solution, is especially related to dynamic axonal relationships of lines, the harmonic relations of the sizes of shapes-forms, the mild or aggressive relationships of materials, colours and lighting" (Haralabidou, 2012)

The experimentation is evolving along with critical thinking about the functional and aesthetic view of space. The consistency of the result with the original intention is important...

As origin of any design miracle, is the formation of the main conception, which the plans become finalized from, acquire personality, and become accepted.

3.4. Finally, for design creativity

Geometry is a structure of the human brain, but if we observe nature we can consider it as the composition of all the laws that surround man. The human rationality was able to recognize the development process in the plant and animal kingdom of some simple forms, to find the relationships that govern these types, with each other, and construct mathematical geometric imaging systems.

Observing nature, we identify and isolate parts of the complex reality of physical structures at different scales, e.g. the proportions of the human body, the symmetry of the flowers and the position of leaves in a branch, up to the spiral of DNA (Thompson, 1999).

In architecture, the process is reversed and becomes immediate, because we use the geometry to build an architectural body. However, as we can not assume that a body in nature is simply geometric, thus, it is not possible to say that geometry is adequate to create - compose an architectural body. The salts e.g. in nature is a construction, not a living entity- organism, because obviously the definition of the word "organism" refers to more complicated forms of life. However, the geometrical arrangement of the form of crystals of salts is directly related to elements, leading to architecture, although its forms are much simpler than those of spatial constructions.

Each architectural ensemble is a complex "body". A plant or animal organism occurs infinitely more complex and complicated. Yet, try to consider:

- "the more a designed form is decomposed to basic geometric forms, the more remarkable it is" is misleading, taking into account that "truth" is a complex event as it relates and refers to humans. A typical example is the Pyramids of Egypt, whose although the external form is basic and simple geometric shape, as forms, they are far from simple synthetic evolution.
- In contrast to the above, we should not believe that complexity of shapes and forms in interior spaces indicates an advanced, sophisticated and modern design. If the synthetic process is not proceeding consciously with the necessary monitoring of any parameters, we risk producing no design.

Aristotle said: *Nature aims to contrasts, and through such dissimilar thing, it produces "agreements". It combines the male and female, and then a union emerges, through the synthesis of opposites.*

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