

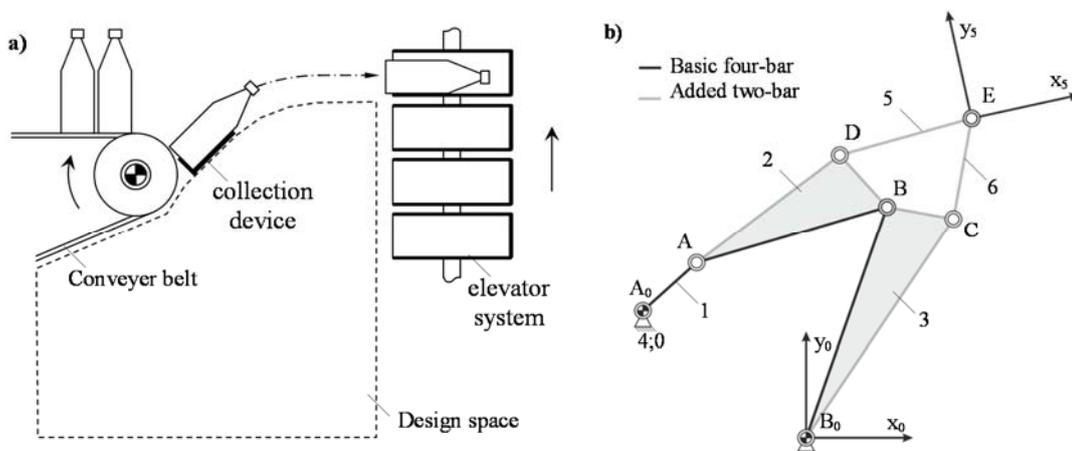
## PARAMETER-DRIVEN MECHANISM SYNTHESIS IN CAD

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*Keywords: mechanism synthesis, modular graphical approach, CAD*

### 1. Introduction

The use of powerful CAD-Systems has become common practice in the design of machines and mechanisms. Exceeding the task of creating the individual parts and assemblies, these systems provide additional supporting modules for quick and efficient analysis of for instance material strength (FEM) or structural dynamics (MBS). However, the iterative process of finding the optimal part dimensions while considering the predefined requirements still is time consuming and involves the creativity and know-how of the development engineer. To expedite the process, interactive synthesis methods for optimal design can be applied at an early stage and integrated into the CAD-System. This is demonstrated in a practical example for the development of a bottle-handling mechanism (Figure 1a).



**Figure 1. a) Bottle conveyor mechanism b) six-bar concept mechanism**

At the end of a conveyor belt, the bottles are placed into a collection device. From there, a mechanism transports the bottles into an elevator system, which moves them to the following production step. Currently, the bottles are pushed up a ramp to reach the elevator.

The new development is to replace the ramp with a linkage mechanism. To this purpose a six-bar mechanism concept (Figure 1b) is selected, which can achieve the required poses. A simpler four-bar mechanism could not be made to fit the design space.

Two types of synthesis methods are used for the development of the mechanism. The dead-centre position synthesis results in a basic four-bar linkage with a revolving crank, which can be connected to the system's main drive. Subsequently, the 3-position synthesis is used to determine the dimensions of the two remaining links (added two-bar) and their connection to the four-bar linkage.

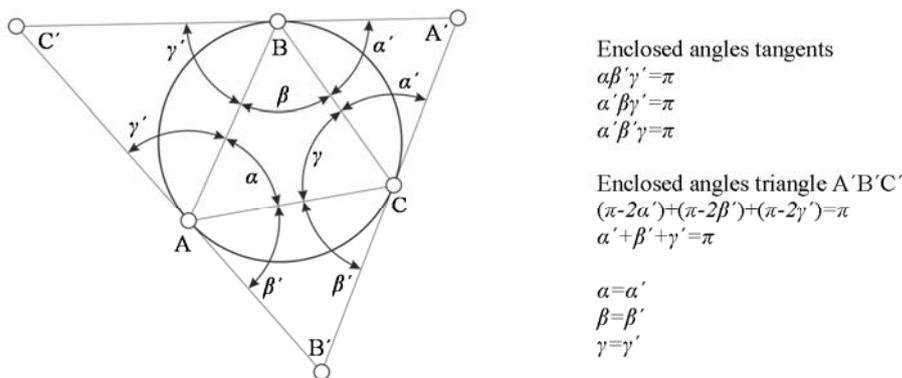
The two methods are explained in more detail in the following chapters. Subsequently the mechanism is created in the CAD-System Autodesk Inventor. Using the synthesis methods in a modular approach several parts are created and combined in an assembly, in which sets of output parameters are passed from one part to the next [Häg11]. With this approach the development engineer is able to optimize the mechanism by focussing on a limited number of parameters whilst compliance with the predefined requirements is guaranteed. Changing the parameters is achieved by dragging elements of the models across the screen, immediately followed by a change of all mechanism dimensions.

### 1.1 Fundamentals

Linkage mechanisms are often used to transform a continuous rotary input motion into an oscillating rotary or linear output motion. During the oscillating output motion the direction of the rocker or slider's motion is reversed. The driven link's positions at which this reversal occurs are known as dead-centre positions.

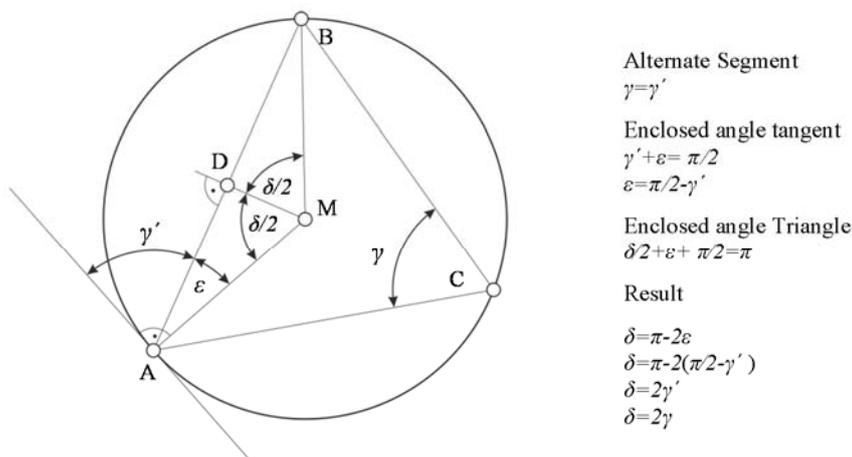
In these cases the accurate kinematic mechanism dimensions must be determined for a given output angle or stroke as well as time ratio for leading and return motion.

An effective approach is provided by a graphical procedure based on the following theorems [Birkhoff 2000]. The alternate segment theorem states that an angle between a tangent and a chord through the point of contact is equal to the angle in the alternate segment (Figure 2).



**Figure 2. Alternate segment theorem**

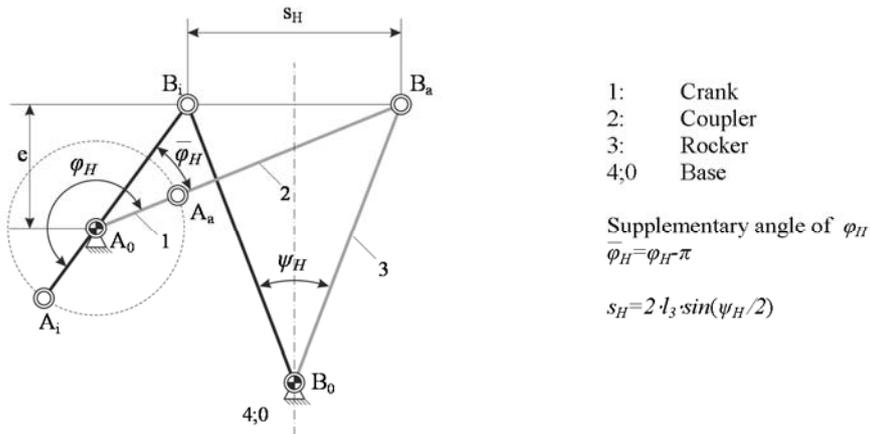
Using the chords (AB, BC, AC) and the tangents (A'B', B'C', A'C'), which form a triangle (A'B'C'), it is shown that the angles in the alternate segments are equal ( $\alpha=\alpha'$ ,  $\beta=\beta'$  and  $\gamma=\gamma'$ ). The arrow theorem states that the central angle ( $\delta$ ) subtended by two points (A, B) on a circle is twice the inscribed angle ( $\gamma$ ) subtended by those points (Figure 3).



**Figure 3. Arrow theorem**

With chord AB, the tangent through A and the lines through the circle center point (AM, BM), using the alternate segment theorem, it is shown that the central angle is twice the angle enclosed by chords AC and BC ( $\delta=2\gamma$ ).

The common input parameters for the dead-centre position synthesis procedure of an arbitrary crank-rocker mechanisms (Figure 4) are the stroke ( $s_H$ ) of joint B, the input angle for the crank ( $\varphi_H$ ) to move the rocker from the outer dead-centre position ( $B_a$ ) to the inner dead-centre position ( $B_i$ ) [McCarthy 2000], [Kerle 2007]. A further kinematic parameter (e.g. the eccentricity  $e$ , the length of the crank ( $l_1=l_{A_0A}$ ) the length of the coupler ( $l_2=l_{AB}$ ) the length of the base ( $l_0=l_{A_0B_0}$ )) subsequently specifies a single solution.

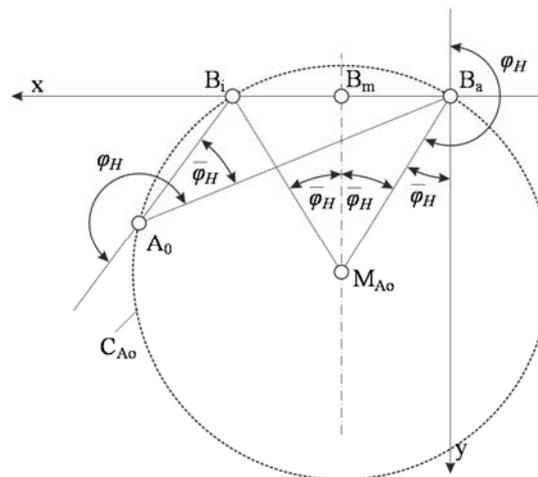


**Figure 4. Crank-rocker mechanism in dead-centre positions**

The stroke ( $s_H$ ) can be calculated using the rocker length ( $l_3=l_{B_0B}$ ) and the swing angle ( $\psi_H$ ). The input angle for the crank ( $\varphi_H$ ) can either be restricted by geometric conditions or is a result of the time difference between the forward and return motion of the rocker (eq. 14) with a constant angular velocity of the crank.

$$t_f/t_r = \varphi_H / (2\pi - \varphi_H) \quad (1)$$

The application of the theorems to the synthesis procedure is shown in Figure 5. The provided dead-centre positions of B ( $B_a$  and  $B_i$ ) define the chord of a circle. The centre point  $M_{A_0}$  is defined by the perpendicular bisector of the chord and the line through  $B_a$  with an the angle  $\bar{\varphi}_H$  relative to the y-Axis of the depicted coordinate system. Any chosen point on the large arc of circle  $C_{A_0}$  provides a solution for the position of base joint  $A_0$ .



**Figure 5. Arrow theorem applied to the dead-centre position synthesis**

In the next step the kinematic dimensions the crank ( $l_1=l_{A_0A}$ ) and the coupler ( $l_2=l_{AB}$ ) (Figure 3) must be defined. For an arbitrary solution of base joint  $A_0$  (Figure 6) the dead-centre angles ( $\beta_i$  and  $\beta_a$ ) between the coupler and the x-Axis of the depicted coordinate system as well as the distances between the base joint  $A_0$  and the dead centre positions of B ( $B_a$  and  $B_i$ ) are reviewed.

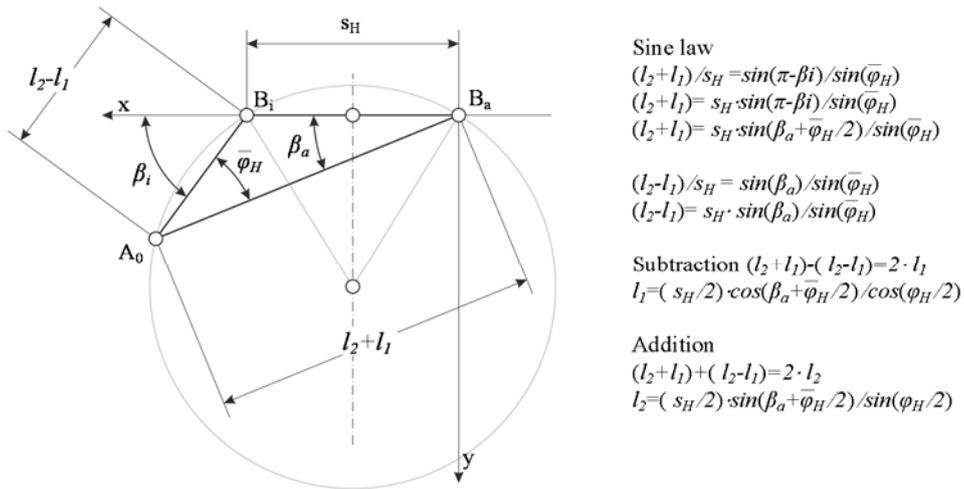


Figure 6. Kinematic parameters

In the inner dead-centre position the crank and coupler overlap. The distance between the base joint and the position of  $B_i$  must be the difference of the length of both parts ( $l_{A_0B_i}=l_2-l_1$ ). In the outer dead-centre position the crank and the coupler assume an extended position. Therefore the distance between the base joint and the position of  $B_a$  must be the sum of the lengths ( $l_{A_0B_a}=l_2+l_1$ ). Using the sine law expressions for  $l_1$  and  $l_2$  can be ascertained. However, the angle  $\beta_a$  remains unknown in these equations. A closer look at these equations reveals the similarity to the common circle equation (Figure 7).

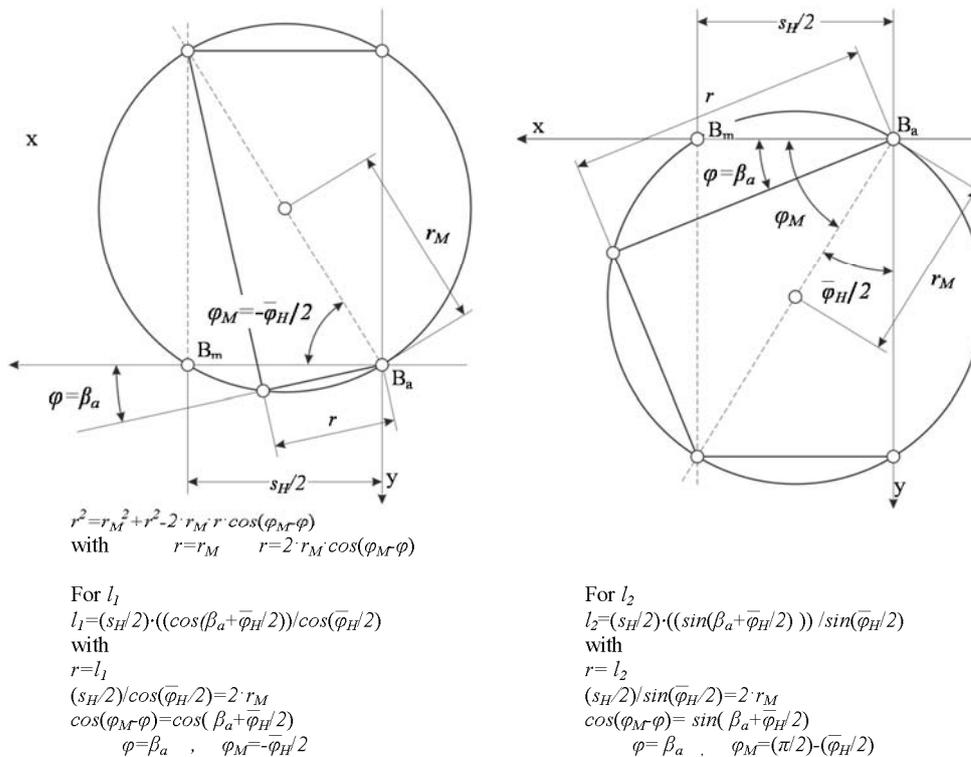
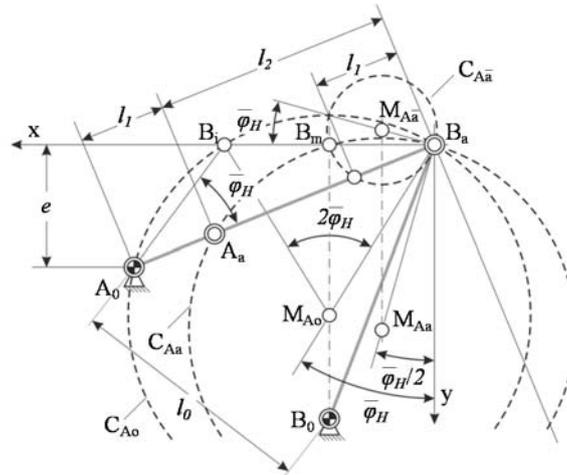


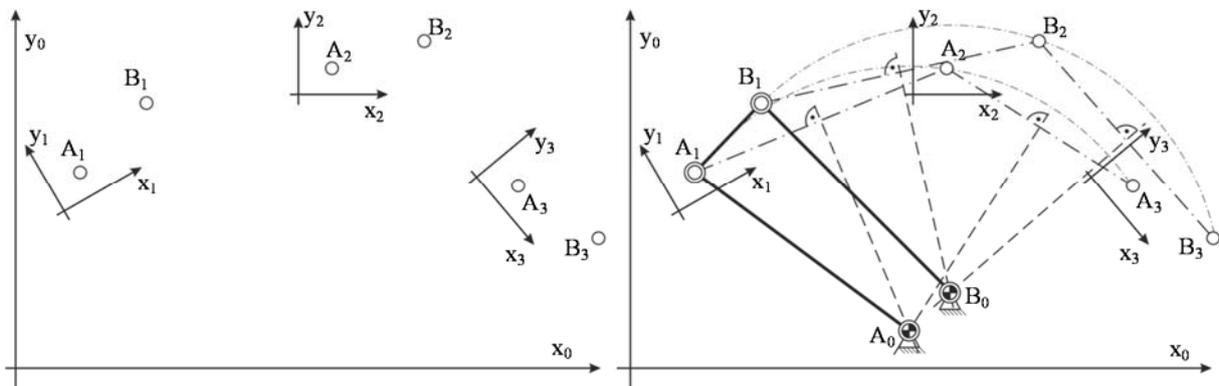
Figure 7. Similarity to the circle equation

Integrating both circles in the dead-centre position synthesis provides the complete set of dimensions (Figure 8) for the crank rocker mechanism. Defining either  $e$ ,  $l_0$ ,  $l_1$  or  $l_2$  will result in a single solution for the kinematic parameters of the crank-rocker mechanism.



**Figure 8. Dead-centre position synthesis**

A second procedure to find the kinematic parameters of linkage mechanisms is provided by the solution of the 3-position synthesis, explained using the example of an arbitrary four-bar linkage. The basis of the procedure can be found in the rigidity of the links, whereby a point on a link maintains its distance to its swivel point. In the first and simpler case the two joints A and B are defined on the coupler and the base joints  $A_0$  and  $B_0$  of the four-bar linkage are unknown. These positions of these joints can be determined by a simple circle-center-construction. The three joints  $A_i$  respectively  $B_i$  ( $i = 1,2,3$ ) lie on a circle whose center is the base joint  $A_0$  respectively  $B_0$ , hence the distance from A to  $A_0$  and B to  $B_0$  remains constant (Figure 9) [McCarthy 2000], [Burmester 1888], [Kerle 2007].

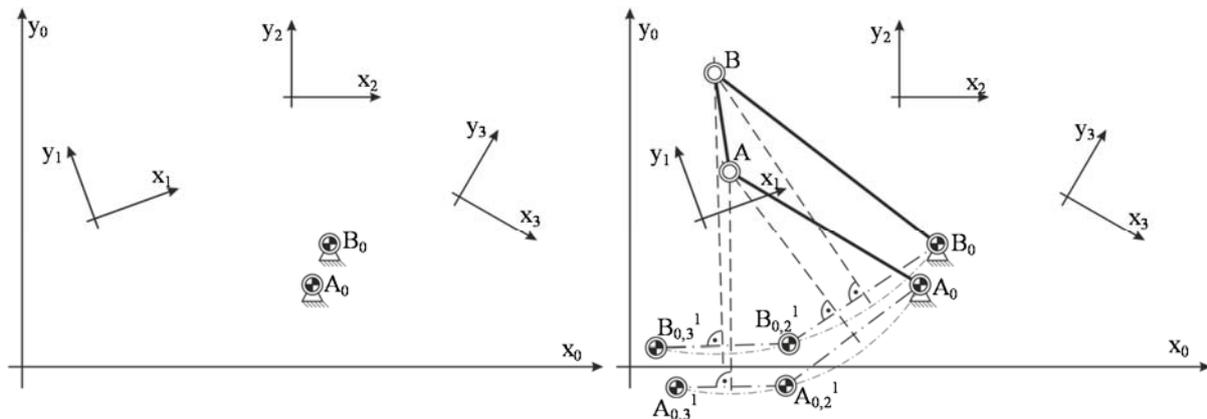


**Figure 9. 3-position synthesis for given coupler joints**

In the second case, the two base joints are defined (Figure 10) and the two coupler joints A and B are unknown. The motion of the base joints relative to the coupler must be circular as well, because the distance of A to  $A_0$  and B to  $B_0$  is constant. Therefore the circle-center-construction must be created relative to the coupler and the relative positions of the base joints in two coordinate systems ( $x_2y_2$ ,  $x_3y_3$ ) have to be projected in the remaining one ( $x_1y_1$ ), resulting in the relative positions  $A_{0,2}^1$ ,  $A_{0,3}^1$ ,  $B_{0,2}^1$  and  $B_{0,3}^1$  respectively. Together with the existing relative position of  $A_0 = A_{0,1}^1$  and  $B_0 = B_{0,1}^1$  the circle-center-construction can be realized. Thus the positions of the coupler joints A and B are determined in the reference system and the synthesized [Burmester 1888].

The two presented methods of the 3-position synthesis are applicable for mechanisms with a higher number of links. However, a four-bar closed loop kinematic chain must be present within the complete

linkage in order to apply the procedure. Moreover, the base joints  $A_0$  and  $B_0$  shown in Figure 4 needn't to be fixed to the frame inevitably. It can be completely equivalent to arbitrary moved joints which form a four-bar kinematic chain with the two required joints.

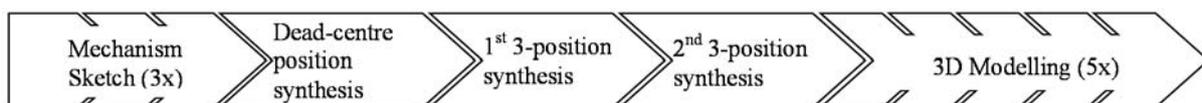


**Figure 10. 3-position synthesis for given base joints**

## 2. Practical implementation

The graphical design procedures can be implemented in any modern CAD systems. In this case, the 3D-CAD System Inventor Professional 2012 was used, which provides a 3D modelling environment and an associative parameterisation. This last feature allows the program to quickly adapt a design to changing parameters provided by the user. The 3D modelling possibilities, for instance the layer arrangement can be used after the kinematic parameters for the six bar mechanism have been found.

To reduce the complexity of the 3D-models, specifically with regard to the implemented design procedures, it is sensible to use a modular approach when creating the CAD-mechanism. This implies that individual models are created for all steps in the process, which are combined to an assembly providing access to the complete process. Because the physical design of the parts is not yet required for the description of design procedures, the models consist of single sketches. Figure 11 shows the process steps.



**Figure 11. process steps in the creation of the mechanism in CAD**

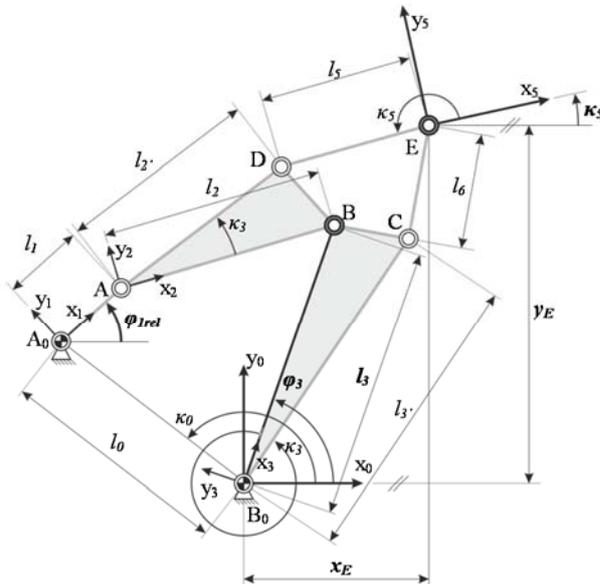
The modular approach also facilitates the general reusability of individual models or rather sketches which can be created much faster than actual parts modelled in 3D.

Essentially parameters are passed from one model to the next within the assembly. However at some point it is necessary to return parameters to a sketch in an earlier stage of the process. These cases would lead to a cyclic parameter transfer which the CAD Program doesn't allow. In Addition, the parameter references must all be created inside the individual models, which frustrates their reuse.

Instead, the parameter transfer can be created inside the assembly using the iLogic programming interface. This approach allows the backward transfer and the control over which parameters are transferred to which model as well as the moment this occurs by defining a rule which triggers this action (e.g. the change of an input parameter). In addition, iLogic allows the user to update the entire assembly after all parameters have been transferred, making the operation of changing a parameter as simple and smooth as possible. The code used in the iLogic can be reused as well.

## 2.1 Parameterization

To start off, the mechanism is created as a rough sketch inside the first model. At this point many of the dimensions provided are only temporary. Only the dimensions gathered from the requirements regarding position of the output link, the length and base joint and position of the rocker are defined. With this in mind the parameters can be subdivided in input and output parameters (Figure 12).



Output parameters defined by the task description and design requirements

Position of the output link

- $x_E$ : Horizontal position of joint E
- $y_E$ : Vertical position of joint E
- $\kappa_5$ : Deflection angle of coordinate system 5

Position of the rocker and joint B

- $\varphi_3$ : Deflection angle of coordinate system 3
- $l_3$ : Length of the rocker

Position of the crank

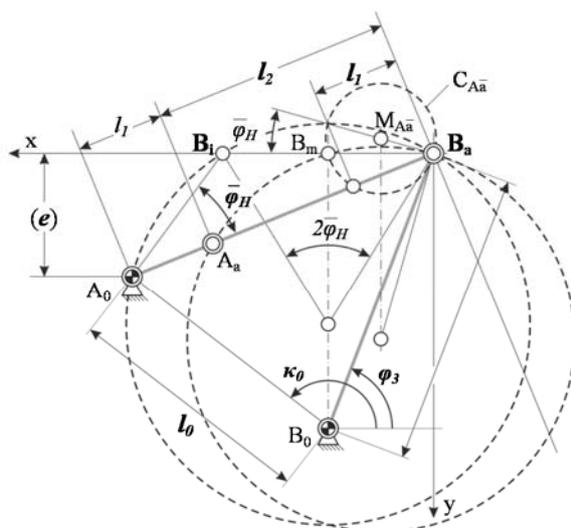
- $\varphi_{1rel}$ : Relative deflection angle of coordinate system 1. (The relative angle is required for the dead-centre position synthesis procedure. The absolute position of the crank is defined by the link lengths  $l_1$  and  $l_2$  which are a result of the procedure)

Figure 12. Sketch model

All other dimensions, respectively the input parameters are defined in the other modules and then set in the sketch model. As Figure 10 indicates the sketch model is integrated three times for three individual configurations of the mechanism.

## 2.2 Dead position synthesis

To provide the dead-centre position synthesis with parameters for the position of joint B ( $B_a$ ,  $B_i$ ), two mechanism configurations must be considered hence two sets of parameters are required (Figure 13). In addition, a selection parameter is required, defining which geometric parameter is used in the procedure to find the undetermined dimensions.



Input parameters

Position of  $B_a$  and  $B_i$

- $\varphi_3$ : Deflection angle of coordinate system 3 for the first and third mechanism configuration
- $l_3$ : Length of the rocker

Parameter choice

- $pSel$ : Selection of parameter
- $e$ : Eccentricity of the base joint position /
- $l_1$ : Desired length of the crank /
- $l_2$ : Desired length of the coupler

Output parameters

Links 1 and 2

- $l_1$ : Synthesized length of the crank
- $l_2$ : Synthesized length of the coupler

Base joint  $A_0$

- $l_3$ : Synthesized length of the base
- $\kappa_0$ : Angle of the x-Axes to the base

Figure 13. Dead-centre position synthesis model

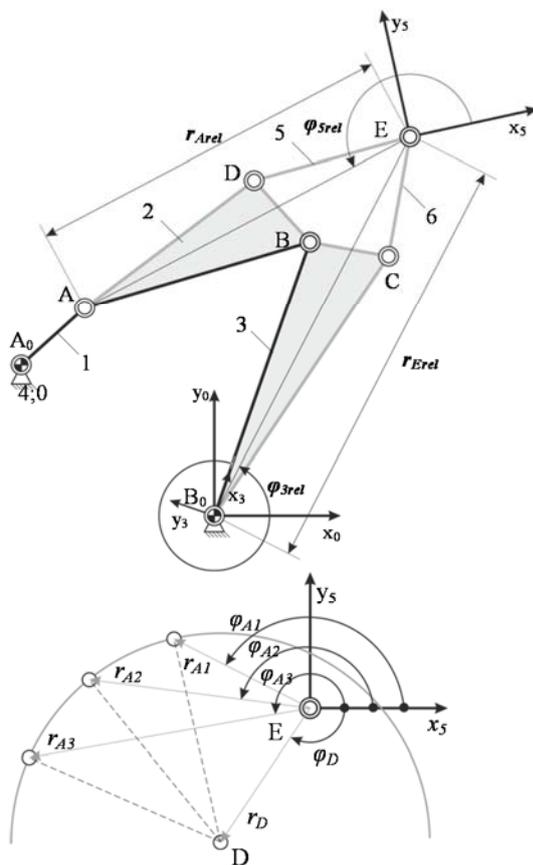
The output parameters are returned to all three mechanism sketch models to set the appropriate mechanism configurations for the following 3-position analysis.

### 2.3 3-position synthesis

The positions of the two remaining revolute joints C and D on the two ternary links 2 and 3 (Figure 1) must be determined such that the guidance task defined at the onset of the development can be achieved. The dead-centre position synthesis provides the kinematic dimensions of the basic four-bar mechanism. Hence, together with the three positions of the rocker, defined in the requirements, the positions of the coupler are defined. In addition, the three positions of the output link 5 (Figure 1) complete the information necessary for the 3-position synthesis.

The models for both 3-position synthesis procedures are identical and require the three positions of a point relative to a reference system.

For the determination of the location of joint C the rocker  $B_0B$  serves as the reference system. The relative positions of joint E must be transferred to the synthesis model (Figure 14). For the determination of the location of joint D the output link serves as the reference system. The relative positions of either joint A or B must be transferred to the synthesis model. Which joint is selected is of no consequence for the result of the procedure [Kerle 2007].



output parameters for the determination of joint D  
 $r_{Arel}$ : Relative distance between joint E and A  
 $\varphi_{Arel}$ : Relative angle form the reference system x-Axis to line EA

output parameters for the determination of joint C  
 $r_{Arel}$ : Relative distance between joint E and  $B_0$   
 $\varphi_{Arel}$ : Relative angle form the reference system x-Axis to line  $B_0A$

The parameters are gathered for each configuration of the mechanism using the three mechanism sketch models

input parameters  
 mechanism configuration 1  
 $r_{A1}, \varphi_{A1}$ : Relative polar coordinates of A  
 mechanism configuration 2  
 $r_{A2}, \varphi_{A2}$ : Relative polar coordinates of A  
 mechanism configuration 3  
 $r_{A3}, \varphi_{A3}$ : Relative polar coordinates of A

output parameters  
 $r_D, \varphi_D$ : Relative polar coordinates of D

**Figure 14. 3-position synthesis for the joint D**

The joint synthesis of C works completely analogously. Unlike the situation for the joint synthesis of D the relative positions of the coupler 5 are not required in this case. The three positions of rocker  $B_0B$  and the three positions of point E are used to solve the 3-position synthesis [Kerle 2007].

### 3. Assembly

Finally, all measurements from the dead-centre position and 3-position synthesis problem are transferred to part models for all five moving links and the base link (Figure 15). The dimensioning of the part models is done equivalently to the dimensioning of the sketches. In addition, other dimensions, such as bearing dimensions and material strengths, can be defined in dependence of the kinematic parameters to further automate the development of the mechanism.

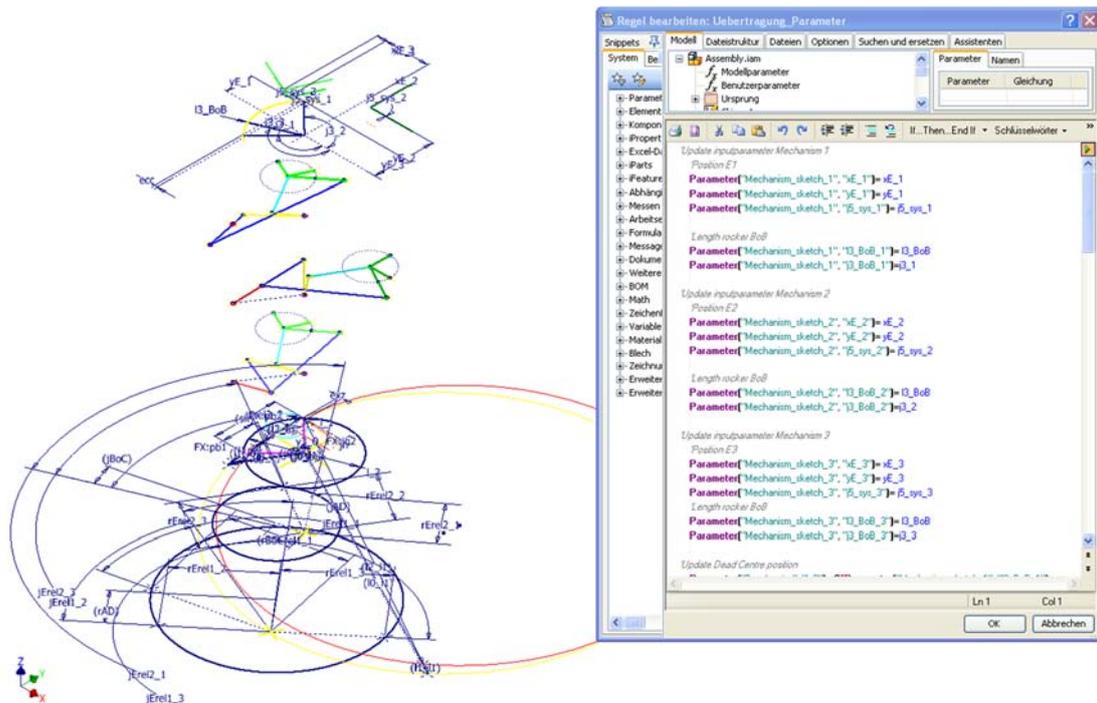


Figure 15. Layered CAD assembly including synthesis procedures, parameter and mechanism sketches

### 4. Summary and outlook

The suggested approach for the development of the six-bar mechanism enables the engineer to interactively adapt a mechanism which comply with the predefined requirements using synthesis procedures. The modular setup allows a reuse of the individual models including the code used in the iLogic. The benefit of using the synthesis procedures inside the CAD system ability of viewing the results of changing parameters enhancing the understanding of the mechanism and how changes can affect its performance.

A next step, further enhancing the approach is the optimization with regard to joint forces, dynamics, vibrations etc. To this purpose, 3D models are created using the kinematic dimension found in the synthesis procedures. Input parameters are subsequently varied within the defined scope, until the kinematic dimensions take optimum values according to set optimisation criteria [Scherer 2011].

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