EVOLUTIONARY DESIGN SYNTHESIS COMPARISON: GROWTH & DEVELOPMENT VS. FIXED-MESH CELLS

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ABSTRACT
Two design synthesis techniques, both utilizing evolutionary computation, are compared. One employs growth and development of a model composed of finite element cells; the other employs a pre-defined fixed mesh of finite element cells. Both of these processes use a similar set of rules. In the fixed-mesh case, some of the rules present in the growth & development case have been disabled to preserve the fixed mesh. The development process starts with a single cell while the fixed-mesh method starts with a predefined volume of material comprising 640 cells. The fitness of individuals in both approaches is determined by evaluating a combination of four different performance objectives: a desired resonant frequency; an ability to support a stochastically varying external load; a desired vertical height; and a minimum weight. A solution with both types of models is found in a relatively small number of generations. However, the growth & development approach generated design configurations with valuable engineering characteristics, including symmetry, smoothness, and regularity, while the fixed-mesh approach generated non-smooth configurations with porosity and inhomogeneity.

Keywords: genetic algorithm, indirect encoding, evolution, development, growth, embryogenesis, stresses, inhomogeneous structure, finite element, artificial cell

1 INTRODUCTION
The development of design synthesis methods has been an active area of research for some time [1]. The goal of this work is to generate novel sets of design configurations that exhibit high performance properties for problems difficult to address with conventional synthesis methods. Design synthesis begins without a proposed configuration, and exclusively utilizes the desired performance of the resulting designs to guide the generation of novel design configurations. Many such techniques utilize stochastic search methods, such as genetic algorithms. This paper presents a comparison of two approaches to design synthesis. The first approach uses a fixed geometric mesh of elements and the stochastic search works to identify the material properties of each cell that produces the best overall performance. The second starts with a single cell that contains a set of growth & development rules; the stochastic search works to identify the set of rules that will cause the structure with the best overall performance to be grown & developed.

1.1 Prior Work
Artificial evolution, in the form of Genetic Algorithms (GA's), has been used in a wide variety of application areas [2, 3, 4, 5]. The goal of this prior work has been to improve one or more features of solutions to a problem. The key element of GA's, first stated by Holland [6], is *implicit parallelism.* The idea is that the GA scheme of actions; selection, crossover and mutation performed on N individuals, implicitly searches for an optimum in N^3 space. This result is powerful, since it enables the rapid exploration of large solution spaces.
The majority of optimization problems that make use of the genetic algorithm approach have employed *direct* encoding. In direct encoding, there is a one-to-one relationship between the genetic information in an individual and the configuration of the individual. Most commonly, the genetic information contains a description of the individual, in contrast with *indirect* encoding, where the genetic information contains a set of rules that, when executed (and perhaps influenced by various environmental factors), guide the growth & development of a single cell into an adult.

Genetic algorithms have been previously applied as an optimization method in structural evolution [2, 7, 8, 9]. Traditional structural evolutionary methods generally start by generating a fixed mesh grid (like a chess board) with a predefined volume and constraints, such as external forces and boundary conditions [10]. Every cell in the grid can have one of two states, either material is present or absent. The genetic information in this approach contains information indicating the material state in each cell of the grid. Once the initial configuration and the boundary conditions are defined (loads, constraints, *etc*.), a finite element analysis is performed, treating each cell as a finite element, and an evolutionary process searches for the configuration exhibiting the best performance. This is an evolution using a *direct* encoding, with no growth or development or embryogenesis, and thus there is a one-to-one mapping between the genetic information (genotype) and the resulting individual (phenotype), resulting in an optimization problem with a large but finite number of possible states [11]. This method produces structures that reflect the underlying shape of the grid [2] and is not able to create continuous or smooth structures. These grid-based building-block structures are not only unrealistic from an engineering perspective, but from a mathematical point of view they search in a limited solution space, resulting in local optima.

An early contribution in this area [2] begins with engineering requirements and subdivides a given spatial domain into a fixed number of finite elements with well-defined boundary conditions. The optimization procedure finds the optimum structure by removing the lightly loaded elements leaving only those which form the optimum structure.

A different approach [7, 8, 9] demonstrates a way to synthesize a compliant two-dimensional MEMS device by eliminating cells from a fixed mesh according to the evaluation of three parameters. From a mathematical point of view, both these methods find the optimum configuration from a finite set of configurations (large but still finite) determined by the initial mesh. This restriction dramatically limits the ability of each method to find a global optimum. Since, for a given design problem, the total number of possible configurations is generally infinite. Another disadvantage of using a fixed mesh is the inability to generate arcs and curved objects without using a large number of elements.

Indirect encoding, and the development of artificial embryogenesis, has been proposed previously [12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. In most examples the goal has been to evolve and grow a predefined target shape on a predefined grid starting from a single cell, using simple rules such as cell division and protein diffusion. Early work in this area has demonstrated the ability of indirect encoding to produce modular phenotypes in graphs and patterns [22, 23, 24]. Advantages of this approach include that the space of possible configurations is infinite and the elements are not restricted to a single shape but are allowed to be deformed and differentiated. In this way smooth, curved, structures can be synthesized.

In the growth & development approach used here, an artificial embryogenesis of structures has been created. This is an extension of previous *indirect* encoding work by the authors [25, 26, 27, 28, 29]. The two fundamental elements of this work are the selection of the artificial cell (the basic structural finite element) comprising each individual, and the artificial genes (the rules) which are evolved into the genetic information of each individual. The genetic information of an individual is shared by all of its cells. Beginning with a single cell, each individual cell executes its rules until a mature structure is formed. Once maturity is reached, an evaluation scheme determines the fitness (performance) of the structure. Evolutionary operations (selection, crossover, and mutation) alter and refine genetic information in a population of individuals over multiple generations. The results are structures that meet the desired performance goals. Computational details are presented in [25], and are briefly summarized below.

1.1 Research Approach

Stochastic methods are increasingly being used to generate, or synthesize, novel design configurations in a range of application areas. This work compares two stochastic computation approaches. Evolutionary computation employing a *direct* encoding has been used previously, as outlined above. *Indirect* encoding, by contrast, is relatively new, and this paper presents a comparison of these two methods. The approach presented here is to establish a design synthesis problem, where no initial configuration is proposed and only the desired performance of the resulting designs is provided, and to employ the two methods and compare the results.

2 DESIGN SYNTHESIS METHODS

The two design synthesis approaches compared here share some methodological characteristics. Both use an evolutionary approach where a population of individuals, each with a unique set of genetic information, is subjected to genetic operations (crossover and mutation) as well as evaluation of performance and selection. In the growth & development process, the genetic information is a set of rules governing the growth and development of an initial single finite element cell into a mature adult. In the fixed-mesh process, the genetic information is the material characteristics of each cell in the mesh (and therefore is a description of the design configuration).

2.1 Rules and Actions

Mimicking nature, the basic structure of a gene is an *if*-conditional *then*-action rule. During the natural embryogenesis of organisms like plants, every 3-D region can deform according to nine different geometric operations: one for isotropic growth, two for anisotropic growth (B), three for shear (S) and three for rotation [30], illustrated in Figure 1. In the artificial embryogenesis used here, three geometric operations were used; isotropic growth/shrink and a shear operation. Geometric operations are defined as actions, and every geometric operation is assigned a unique alphabetic letter shown in Table 1.

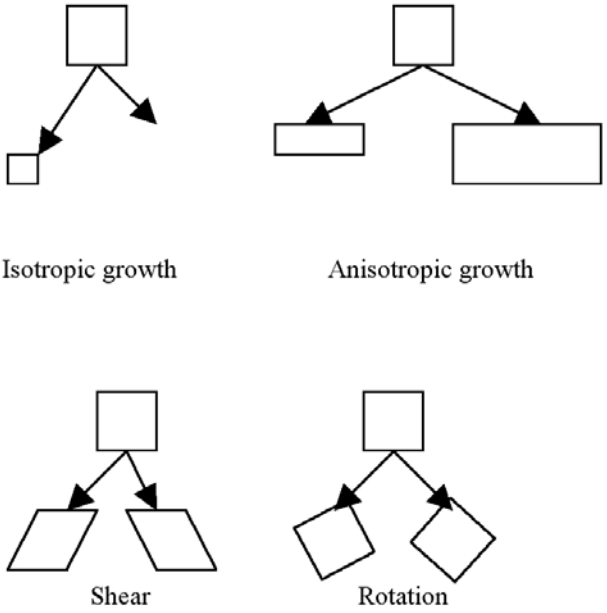


Figure 1. The four basic geometric operations observed in sub-regions of plants.

Table 1. Cell geometrical actions.

ID	Name	N	Parameters
A	Shear	1	(d,e,f,i) × fractional coefficient
S	Isotropic Shrink	3	(a,b,c,g,h) × fractional coefficient
C	Isotropic Growth	1	(a,b,c,g,h) × fractional coefficient

In addition to the geometric operation actions, cell-type actions are defined, shown in Table 2. These actions are the three basic operations that occur in the developmental process of every biological structure, including; cell division, cell death, and cell differentiation. Cell division splits the cell into two equally sized cells, such that the total volume of the divided cells remains the same as that of the initial single cell. Cell death causes a cell to be removed from the model. Cell differentiation alters the material properties of a cell. The properties of the three types of materials that the synthesis methods can utilize are shown in Table 3.

Table 2. Cell-Type actions.

ID	Name	N	Parameters
D	Cell division	1	(*d*,*e*,*f*,*i*)
K	Cell death	0	
F	Cell differentiation	0	(0,1)

Table 3. Cell Materials.

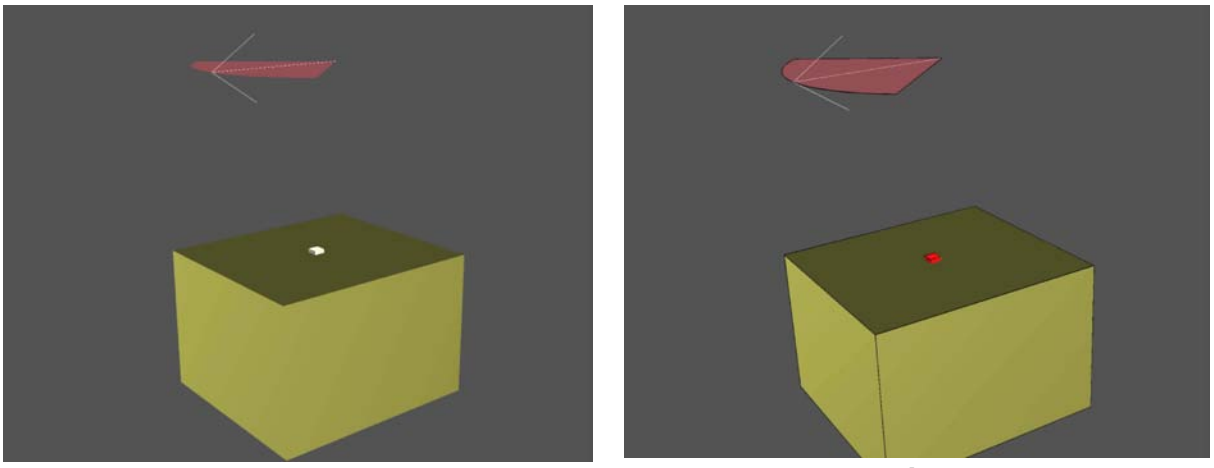
Material ID	Yield Stress [*MPa*]	Young's Modulus [*MPa*]	Density [Kg/m^3]
0	700	240	7 800
1	600	145	5 800
2	340	190	7 400

2.2 Environment

The environment in which the individuals are grown contains factors that every cell can sense and which can affect the way rules are executed, similar to the way genes are expressed. The relationship between the information that cells receive from the environment and the growth & development of the phenotype is not predetermined. Rather, conditionals are available to the evolutionary process that sense the concentration or gradient of each morphogen. In this way, the evolutionary process establishes the relationship between information and growth & development. As the phenotype is being grown, it is evaluated by means of a finite element analysis to determine the pattern of mechanical stresses and deformations in the phenotype [31]. Every cell is an extended 3-D non-orthogonal finite brick element. Therefore, the structure and the mesh are identical, and are evolved simultaneously during growth & development. Cells also maintain information relating to their size, age, and distance from neighboring cells.

2.3 Growth & Development Process

In the development process a single cell grows & develops into a mature adult phenotype. The process starts with a single cell placed on the ground subjected to a gravity field, as shown in Figure 2. The large brown block at the bottom of each image in Figures 2, 3, 5, and 6 is the ground. The *Z* direction is perpendicular to the top surface of the ground. The cell is initially made of material type #0.



a b

Figure 2. Initial state of the growth & development process. a) A single cell initially made of material type #0. b) The stress distribution in the cell in the initial state, generated by the stochastically varying load which is shown by the pink region at the top.

A morphogen radiating from the position of the load is placed at a fixed point in space above the ground. The morphogen diffusion and the gravity field create an environment that is sufficient to regulate the rules in the genome. Once a phenotype has grown to reach the morphogen, a load is

applied to the top cells that coincide with the location of the source of the morphogen. This load generates a mechanical stress distribution along the phenotype which will alter the local environment on the cells, and can cause the rule/gene regulation mechanisms to alter the further growth & development of the cells. The pink region in Figure 2a,b represents the external load variation, at each time step a new load vector in this region is selected. The process of evaluating the mechanical stresses on the cells at each time step is performed by solving a finite element scheme. The load itself is randomly changes its direction and thus the mechanical stress inside the cell may change as well.

2.4 Fixed-Mesh Process

The fixed-mesh process, unlike the growth & development process, starts with a predetermined volume of material comprising a predetermined mesh of cells. All cells initially are made of material type #0, indicated by their brown color as shown in Figure 3a. During the fixed-mesh synthesis process, cells can either be removed or can differentiate (change material type) according to the genetic information in each cell. The fixed-mesh is maintained by utilizing the same set of rules as in the growth & development process, with the exclusion of rules for cell division, growth, and shear operations.[1] Excluding these rules from the fixed-mesh model maintains a fixed cell size and a fixed mesh grid.

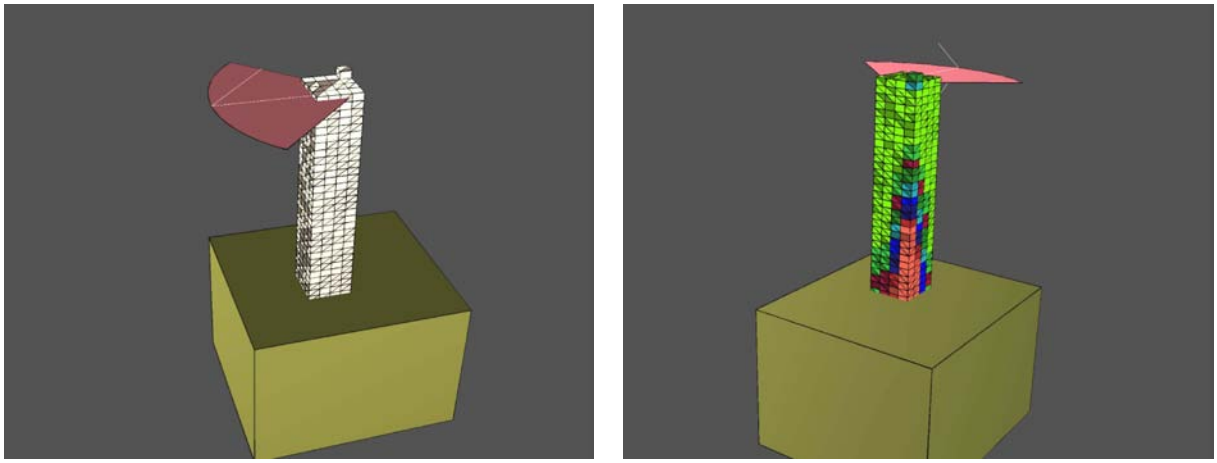


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Figure 3. Initial state of the fixed-mesh process. a) A predetermined block made of 640 cells, all cells are made of material type #0. b) The stress distribution on the cells in the initial state, generated by the stochastically varying load which is shown by the pink region at the top.

2.5 Genome Structure

The genome contains words which contain genes identified by their corresponding letters (shown in Tables 1 through 2, and in more detail in [25]). Every word starts with the letter “R” which indicates the number of times the particular word will be executed in a single time frame. The letter “Z” indicates the beginning of the word. The genes contain operations, parameters (*e.g.*, morphogen concentration or gradient) and coefficients. Similar to transcription factors in nature, coefficients are numbers between zero and one, that scale an effect in proportion to the concentration of the signaling chemical to which they refer.

2.6 Conditionals

The conditional artificial genes are “veto” or “suppression” genes. These genes affect other genes only at the genome level, by turning actions off or on according to whether the conditional test is satisfied or not.

[1] This approach is somewhat different from a typical direct encoding approach, where the genetic operations alter the information in each cell that describes the cell.

2.7 Evolutionary Scheme

The evolutionary scheme is derived from a genetic algorithm with four steps: evaluation, selection, crossover, and mutation, and each repetition is defined as one generation. The algorithm is initialized with a set of randomly generated genomes. Starting from a single artificial cell, one individual is grown from each genome by executing the rules it contains. The fitness of each individual is evaluated by means of the finite element analysis and the aggregation of additional properties, described below.

2.8 Evaluation Scheme

The evaluation scheme is based on an aggregation approach where several parameters which represent the performance of a phenotype are aggregated into a single scalar. Seven parameters are taken into account here: resonant frequency, mechanical stress, weight, shape of the cells, distance from the location of the load, age of the phenotype, and the maximum volume of a cell. The goal is to *minimize* the fitness value, thus lower fitness means better performance. Evaluation of an individual is performed only after the growth & development process is completed. The resonant frequency of the final structure is computed using the power method in order to save computational resources. The mass matrix *M* and the stiffness matrix *K* are constructed as part of the assembly process of the finite element scheme. The matrix *A* is defined in Equation 1 and has two dominant eigenvalues: minimum and maximum. The process of determining the lowest eigenvalue is performed using the recursive relation, shown in Equation 2 where **x** is a vector with initial values set to 1.0. Once the relation in Equation 2 has converged to the minimum resonant frequency, f_{min} is determined by equation 3. This process usually converges in 5 to 10 iterations which makes it highly efficient compared to a full eigenvalue solver.

$$\mathbf{A} = \mathbf{M}^{-1}\mathbf{K} \tag{1}$$

$$\mathbf{x}_{i+1} = \mathbf{A}\mathbf{x}_i \tag{2}$$

$$f_{\min} = \frac{1}{2\pi}\sqrt{\frac{(\mathbf{A}\mathbf{x}_{i+1})\cdot(\mathbf{x}_{i+1})}{(\mathbf{x}_{i+1})\cdot(\mathbf{x}_{i+1})}} \tag{3}$$

2.9 Genomre Execution Process

In nature, the process by which cells express their DNA is stochastic with respect to time. Unlike computers where a central clock synchronizes logic operations, in nature, the expression of genes occurs with respect to conditions known as gene switches [32]. The gene switches create an asynchronous order in which genes are activated during the growth & development process. In the methods compared here, time is discretized into frames. At any instant of time during growth & development, a number of cells *N* exist and contain identical genomes, each with *n* words. The stochastic behavior of gene expression is mimicked by applying a Poisson process to the order of execution of the rules in each genome.

3 RESULTS

Two experiments were run, one for the fixed-mesh process and one for the growth & development process. In each experiment an initial population of 400 genomes was generated randomly. Genetic operations such as selection, mutation and crossover were performed at the genome level at the end of each generation. The desired height of the structure was 7 m, the external load was 10^6 N with random variation in direction over ±30, and the desired resonant frequency was 20 Hz. Cells were able to differentiate into three different types of material, each with different mechanical properties, as shown in Table 3. As mentioned above, the growth & development scheme includes all the types of rules, *i.e.*, operations such as cell type and cell shape, while the fixed-mesh process excludes cell division operations and all operations that change cell geometry.

At every generation the individual with the best performance survives into the next generation. Figure 4 shows a comparison between the best fitness value in every generation for both processes. In both approaches, as can be seen in the figure, the best fitness value at each generation fluctuates from

generation to generation, even though the best individual survives into the next generation. The reasons for these types of fluctuations are that the variations in the environment and the random order of execution of rules within the genome cause variations in the growth & development of individuals. These two stochastic effects do not assure a constant fitness value even for the same genome. For example, a particular structure may support a load in a particular direction, but when the performance of the structure is evaluated for a load in a different direction, still within the expected range (illustrated in Figure 3), the structure may perform more poorly or may fail. The same is true for the genome execution process. A different order of execution of rules within a genome can give rise to different configurations, and therefore different performance.

Figure 4 shows that the growth & development process converged to a solution after 380 generations while the fixed-mesh process converged to a solution after only 50 generations due to the much smaller size of the design space. However, the fluctuation of the fitness value had a higher variance in the fixed-mesh process. In other words, even though the growth & development process required more generations to converge to a solution, the performance of the result is more robust to variations in the environment and during growth and development.

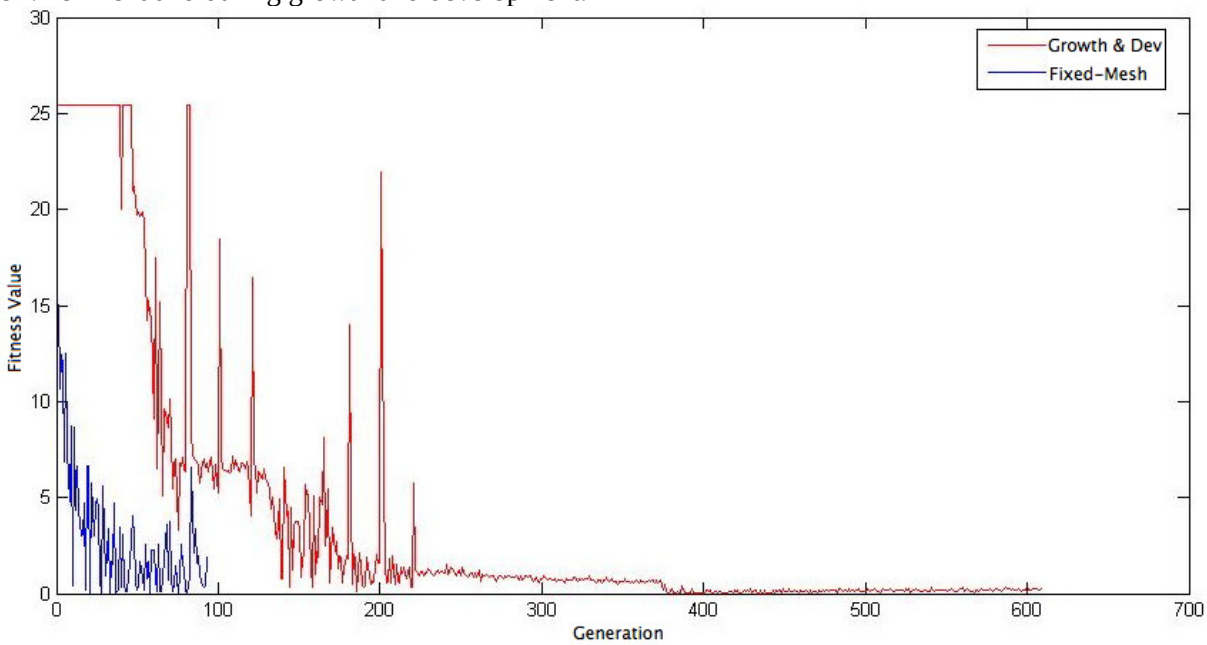


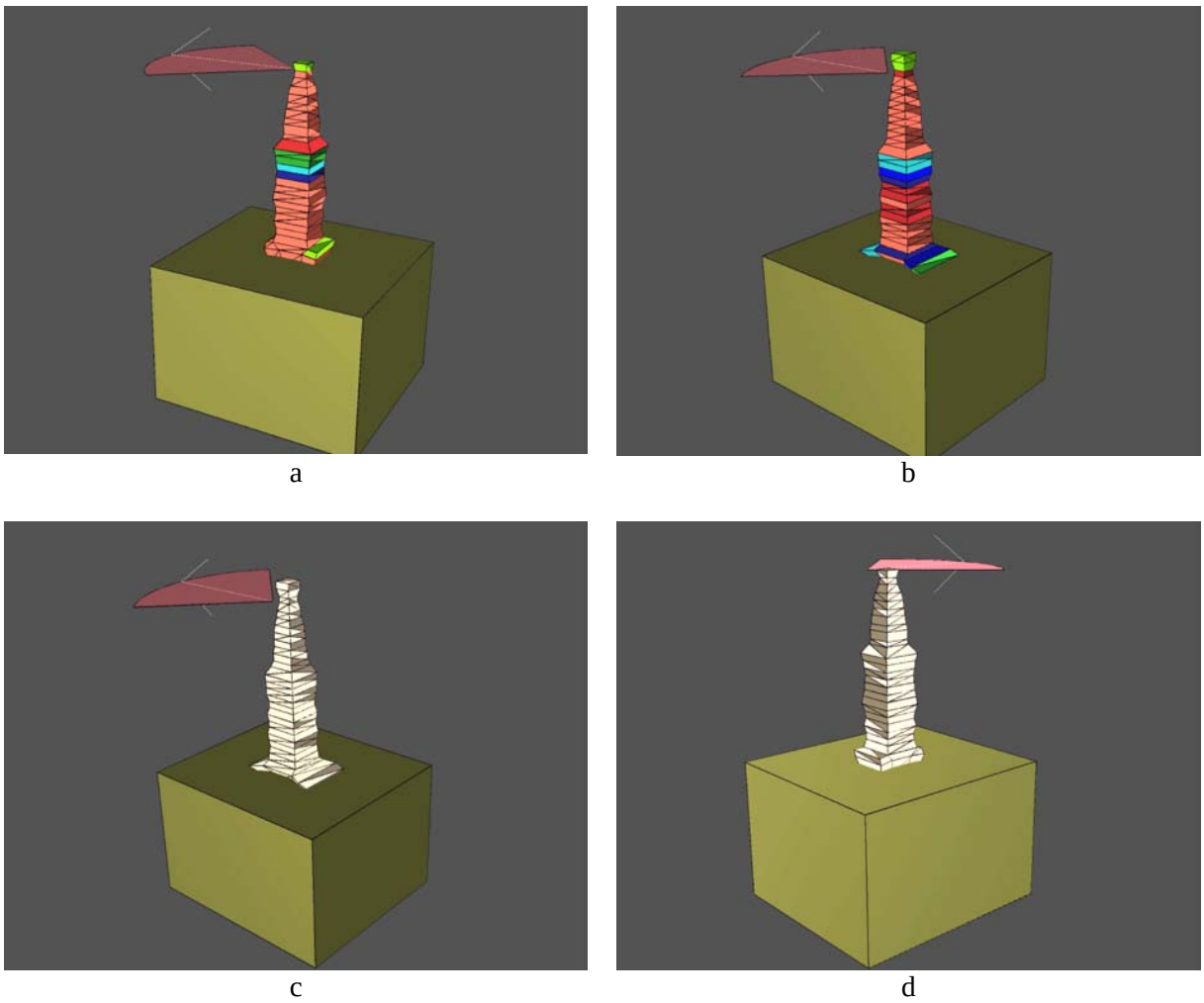
Figure 4. The fitness value of both processes as a function of generation.

Figures 5 and 6 represent two phenotypes selected from the last two generations for both the growth & development and the fixed-mesh processes, respectively. The cells comprising the phenotype in Figure 5 have a range of shape, size and orientation while the cells in Figure 6 retain their fixed cell size over a fixed mesh grid.

The phenotype generated from the growth & development scheme has a regular configuration with three regions along its length. Starting from a wide base at the bottom, transitioning into a mid-column in middle, and finally into a narrow column an the upper end. The mid- and the upper-column, all made from single cells with different volume and orientation, attach to each other along *Z*-direction. The base region contains more cells, attached to each other both in the *Z* and *X* directions. Figure 5 shows the smooth transition between one region to another, which reduces stress concentration. The figure also illustrates that the structure is homogeneous and convex (no voids). The former characteristic indicates that the differentiation mechanism has not been utilized, promoting each one of the objectives (low stress, resonant frequency, *etc*.).

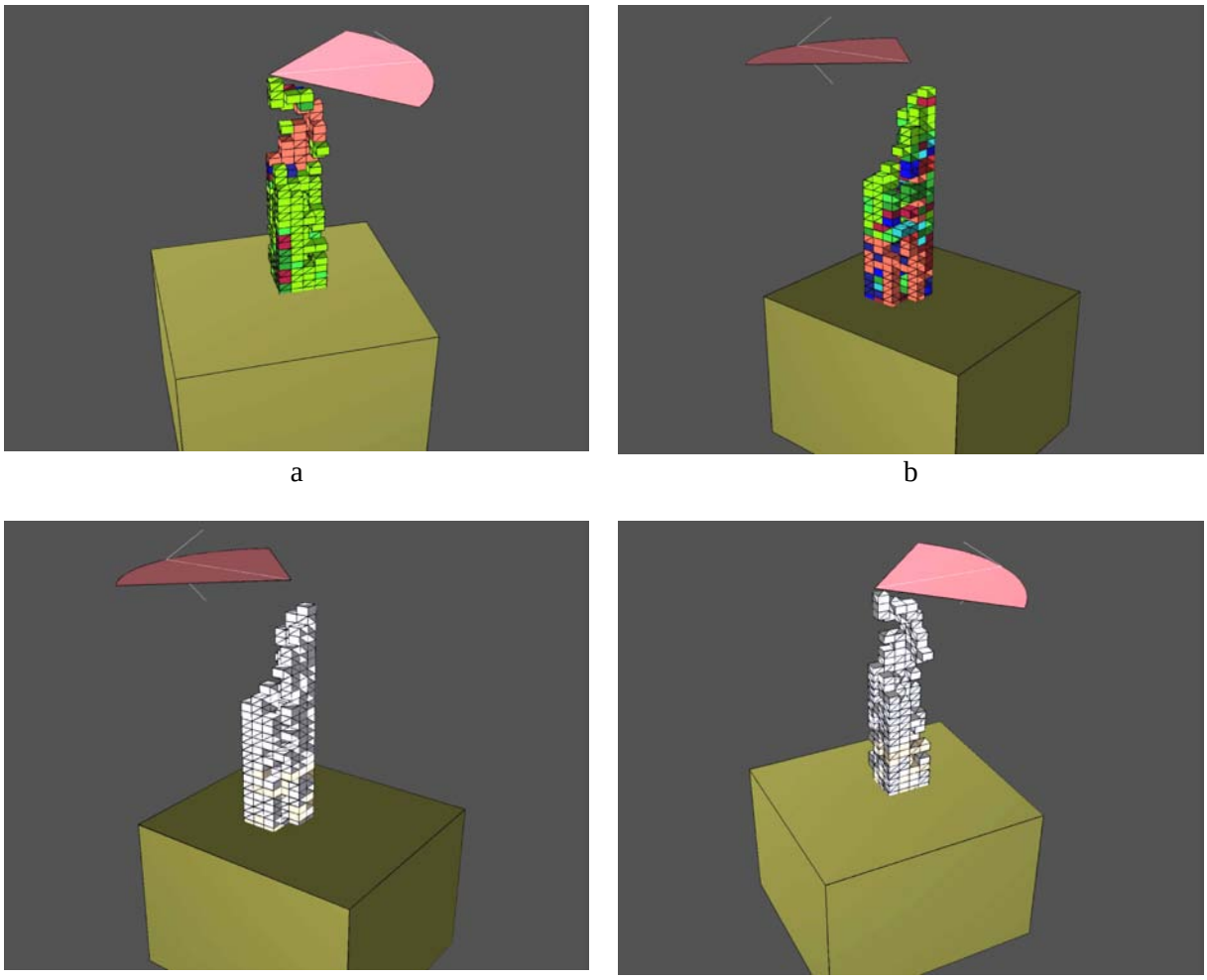
Geometric symmetry can also be observed in this structure. The structure is highly symmetric in the *X*-*Z* and *Y*-*Z* planes. This characteristic has been spontaneously synthesized without being part of the evolutionary fitness evaluation scheme. The two phenotypes, generated from the growth & development process, shown in Figure 5, are almost topologically identical. This indicates that the evolved genome is robust to both environmental effects (*e.g.*, variations in load direction), and the random order of execution of its genes.

The phenotype in Figure 6 comprises single size cells arranged on a fixed-mesh grid. The phenotype has been evolved with respect to the same objectives as the growth & development case, and has achieved a similar fitness value. Figure 6 shows that the structure is inhomogeneous, comprising different material type of cells. The structure also possesses a high degree of porosity. The combination of these characteristics enabled this particular structure meet the performance objectives, and attain a high fitness value. However, neither symmetry nor regularity is present in this structure. Also, the best phenotypes generated from the last two generations of the fixed-mesh process are topologically dissimilar. This type of dissimilarity indicates a lower level of robustness of the evolved genome, when compared to the best genomes evolved by the growth & development process, as shown in Figure 4.



a b c d

Figure 5. Two phenotypes from the last two generations of the growth & development scheme with 20.15 and 20.07 Hz resonant frequency, respectively. a,b) Stress distribution over the cells. c,d) The material type of the cells, brown corresponds to material type #0, gray corresponds to material type #1.



a b

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Figure 6. Two phenotypes from the last two generations of the fixed-mesh scheme with 19.87 and 19.75 Hz resonant frequency, respectively. a,b) Stress distribution over the cells. c,d) The material type of the cells, brown corresponds to material type #0, gray corresponds to material type #1.

4 CONCLUSION

Two design synthesis schemes: growth & development and fixed-mesh processes have been used to evolve the design configurations of three dimensional resonators with a multiple number of objectives: resonant frequency; mechanical stress; weight; and height. In both schemes, a genetic algorithm is used to alter and refine a genome that corresponds to each individual in a population. The growth & development process starts with a single cell which grows & develops into a mature phenotype following the execution of rules in its genome. The fixed-mesh process starts with a predefined volume of 100 cells which are either removed or change material type during the synthesis process. Both schemes achieved similar performance (aggregated fitness) with respect to the objectives. However, the topological characteristics of the phenotypes generated from each of the two processes were significantly different.

The configurations synthesized by the growth & development process contain engineering characteristics such as symmetry, smoothness and regularity, while the configurations generated by the fixed-mesh process are non-symmetric with a high degree of non-smoothness, porosity and inhomogeneity. From a mathematical view, the space of solutions in the growth & development scheme is much larger than that of the fixed-mesh scheme. The former has an infinite number of solutions, since cells can be located and oriented anywhere in the medium while the latter has a finite number of configurations due to the fixed-mesh grid. Nevertheless, a solution has been achieved in both cases in a relatively small number of generations. This result indicates that the growth & development process (embryogenesis through the evolution of rules for growth and development to create a recipe shared by all cells) is a powerful method for synthesizing the design configuration of highly complex structures that must meet multiple, conflicting, performance goals.

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