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APPLICATION OF DAMAGE MODEL FOR NUMERICAL DETERMINATION OF CARRYING CAPACITY OF LARGE ROLLING BEARINGS

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1. Introduction

Determination of the actual carrying capacity of the rolling contact in low speed axial bearings with the established standardised criteria did not give satisfactory results. An exploitation of large axial bearings also includes some load peaks, which cause permanent deformation of rolling contact. The plastic strain of the base material under the hardened rolling layer starts to grow. The consequence may be micro cracks on the edge of the hardened layer as well as pealing-off of the hardened layer. Because of that we have used a constitutive damage model for determining subsurface strain stress relationship. The model includes isotropic and kinematic hardening or softening and growth of damage with growth of oscillation. We have integrated the damage model into a numerical computation with a finite element model where the simulation of the contact of the raceway and the rolling element under a cyclic load has been done. The comparison of experimental and numerical results confirmed the suitability of the established model for the determination of the actual carrying capacity of the rolling contact in low speed axial bearings.

2. Modeling of cyclic plasticity and damage

The existence of microscopic voids or cracks the size of a crystal grain is referred to as material damage. The state of damage in material D is theoretically determined by the overall influence of the size and the configuration of microcracks and microvoids. The actual macroscopic stress within the damaged material $\bar{s}$ is determined by assuming (1) that the nominal cross-section A is reduced by the size of the damaged area A_D [Lemaitre 1990, Lemaitre 1990, Skrzypek 1999]:

$$D(M,\vec{n},x) = \frac{dA_{Dx}}{dA} \quad \Rightarrow \quad \bar{s} = \frac{F}{A - A_D} = \frac{F/A}{1 - A_D/A} = \frac{s}{1-D} \ . \tag{1}$$

The material damage concept is built into the constitutive model of small deformations e_{ij} which are composed of the elastic and the plastic part:

$$e_{ij} = e_{ij}^{\ e} + e_{ij}^{\ p} \ , \tag{2}$$

where the elastic relationship between stress s_{ij}, strain e_{ij}^{e}, damage D and elastic modulus tensor L_{ijkl} is determined by:

$$s_{ij} = (1-D) \quad L_{ijkl} \cdot e^{e}_{kl} \ . \tag{3}$$

The rate of plastic strain is derived from the normality rule:

$$\dot{e}_{ij}^{\ p} = \dot{l} \frac{\partial f}{\partial s_{ij}} \ . \tag{4}$$

where $\dot{l}$ is the plastic multiplier, derived from the consistency condition $\dot{f} = 0$. The stress potential f [Lemaitre 1990] is a function of stress tensor s_{ij}, the components of kinematic (X_{ij}) and isotropic (R) hardening and the material damage D. For an isothermal state dT/dt = 0, the rheological model and the evolution equation of the stress potential are determined by [Lemaitre 1990 and Lemaitre 1996]:

$$f = \tilde{s}_{eq} - (R + k) = 0 \, , \tag{5}$$

$$\tilde{s}_{eq} = \sqrt{\frac{3}{2} \tilde{s}_{ij} \tilde{s}_{ij}} \ , \ \tilde{s}_{ij} = \tilde{s}_{ij} - \frac{1}{3} d_{ij} \tilde{s}_{kk} \ , \ \tilde{s}_{ij} = \frac{s_{ij}}{1-D} - X_{ij} \tag{6}$$

2.1 Kinematic hardening model

Kinematic hardening is described by the back stress tensor X_{ij}, which determines the centre of the yield surface in stress space. To determine the values of each of the three tensor components, the evolution equations proposed by [Armstrong and Frederick 1966], and [Chaboche 1988] have been used. The boundary value of hardening is determined by the dynamic hardening coefficient $X_8^{(n)}$, whilst the value of the plastic extension, at which the respective components $X_{ij}^{(n)}$ reach their boundary value, is defined by the kinematic hardening level $?^{(n)}$. The influence of the reduction rate of the mean stress value is controlled by the Ohno-Wang material parameters m_n [Ohno 1993].

$$X_{ij} = \sum_{n=1}^{3} X_{ij}^{(n)} \quad , \ \dot{X}_{ij}^{(n)} = \frac{2}{3} g^{(n)} X_{\infty}^{(n)} (1-D) \dot{e}_{ij}^{pl,c} - \left(\frac{X_{eq}^{(n)}}{X_{\infty}^{(n)}} \right)^{m_n} X_{ij}^{(n)} g^{(n)} \dot{l} \ \ , \tag{7}$$

where the effective value of the stress space centre tensor is

$$X_{eq}^{(n)} = \sqrt{\frac{2}{3} X_{ij}^{(n)} X_{ij}^{(n)}} \ . \tag{8}$$

2.2 The isotropic model of hardening/softening

The material is subject to cyclic hardening or softening if the yield surface increases or decreases during a cycle loading process. The size of a yield surface is described by scalars R and k. R represents the variable which describes isotropic hardening or softening of the material, while k represents the size of the elastic area. The initial values for the size of the yield surface are $k = s_y$ and $R = 0$, where s_y represents the yield stress. The evolution equation for isotropic hardening or softening R has the following form [Lemaitre 1990 and Lemaitre 1996]

$$\dot{R} = b(R_{\infty} - R) \dot{l} \ \ , \tag{9}$$

where b represents the material parameter, which determines the level of isotropic hardening or softening, while the parameter R_8 defines the boundary of isotropic cyclic hardening or softening. The limit value for cyclic hardening or increased yield surface is $R_8 > 0$ and $R_8 < 0$ for cyclic softening or decreased yield surface.

2.3 Continuum damage mechanics

Irreversible damage growth can be described by the evolution equation (10) [Lemaitre 1996, Chaboche 1988] which considers proportional influence of the effective plastic strain on the change in the damage. The evolution equation, which describes the damage has the following form:

$$\dot{D} = \frac{s_{eq}^{2}\left(\frac{2}{3}(1+n)+3(1-2n)\left(\frac{s_{kk}}{3s_{eq}}\right)^{2}\right)}{2\cdot S\cdot E(1-D)^{2}}\,\dot{p}\cdot a(p) \quad ; \quad a(p)=\begin{cases}1, & if \quad p\geq p_D\\ 0, & if \quad p<p_D\end{cases} \quad . \tag{10}$$

The initial damage threshold p_D is the boundary and is determined by the accumulated plastic deformation p:

$$\dot{p} = \frac{\dot{l}}{1-D}\ , \quad p_D = Max(p_{(D=0)}) \ . \tag{11}$$

3. Computer implementation

The material model has been derived and coded by using hybrid system for multi-language and multi-environment generation of numerical codes. The system consists of two major components: the Mathematica package AceGen that is used for the automatic derivation of formulae and code generation [Korelc 1997], and the Mathematica Computational Templates package with the prearranged modules for the creation of the finite element codes. The Computational Templates package enables the generation of multi-language and multi-environment finite element code from the same abstract symbolic description. The simulation development process can be divided into three characteristic steps (see Figure 1).

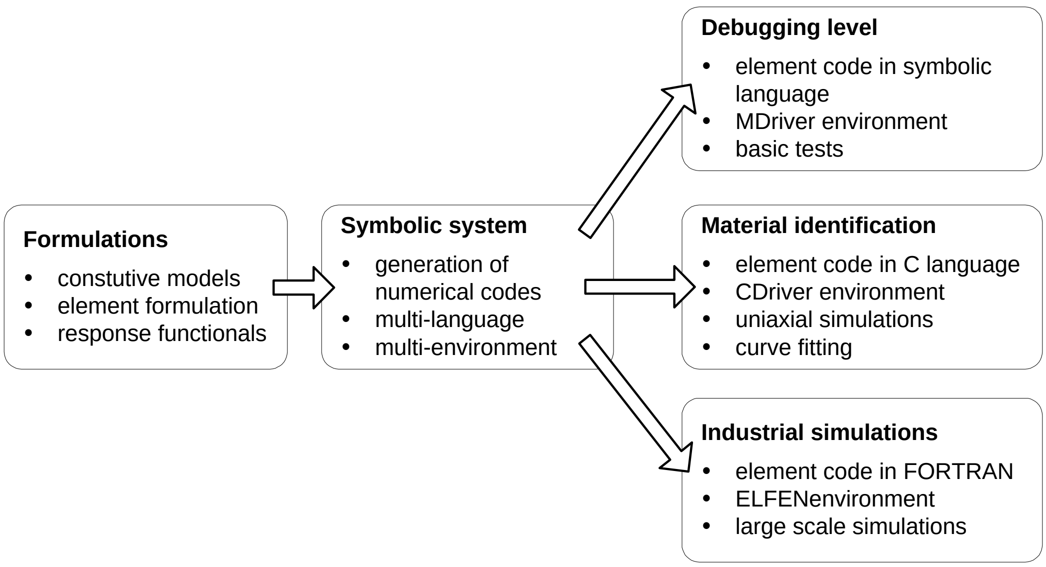

Figure 1. Outline of the code development concept

The basic tests are performed on a single finite element or on a small patch of elements by using a general symbolic-numeric environment Mathematica and the element code written in the symbolic language of Mathematica. For the simulation of uniaxial experiments and identification of material parameters, efficiency and flexibility is needed at the same time. For this purpose the element code written in C language is linked with the symbolic-numeric environment Mathematica. For large-scale industrial cases good mesh generation and an efficient solver are essential, so in the third step, the FORTRAN code was produced and incorporated into commercial finite element code ELFEN.

4. Low cycle carrying capacity for large bearings raceway

4.1 Determining of material parameters

For an established numerical material model we have determined the material parameters of a bearing ring's low alloy steel 42 CrMo 4 with a help of various experimental low cycle tests [Kunc 2001]. Figure 2. shows the comparison of the maximum and the minimum amplitude peaks and the comparison of the hysteresis loops relation to the number of load cycles, obtained from the measurements and the chosen numerical model. All the required parameters of kinematic hardening, isotropic softening and damage growth, which influence the elasto-plastic material response of the 42 CrMo 4 low alloy steel can be accurately determined from as few as 10 experiments ,which proves that simulation of the material response to low-cycle loading resulting in destruction is very reliable (Figure 3 and Figure 4).

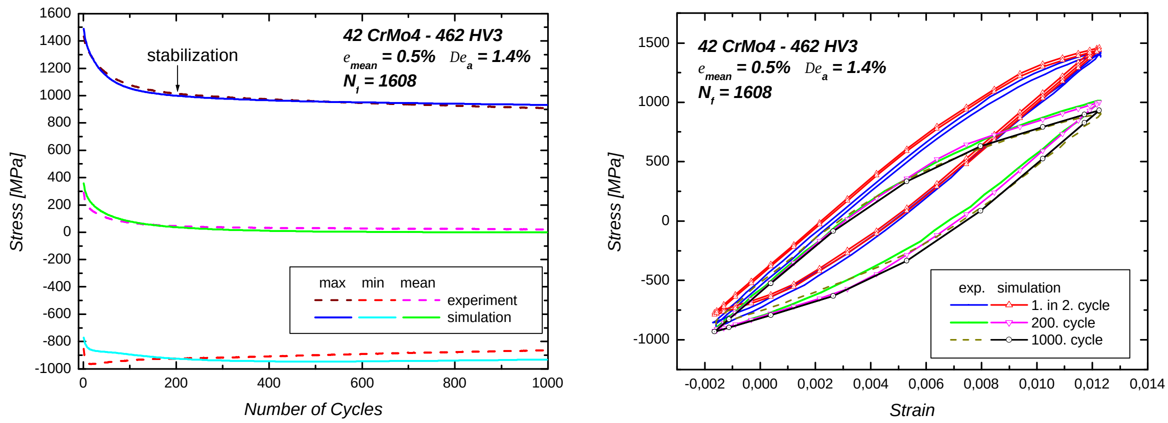

Figure 2. A comparison between measurement and numerical calculation for the tempered steel 42 CrMo 4 with hardness 462 HV. A non-symmetrical load case at a constant extension of ? e = 1.4 % and a mean value of e_{mean} = 0.5 %

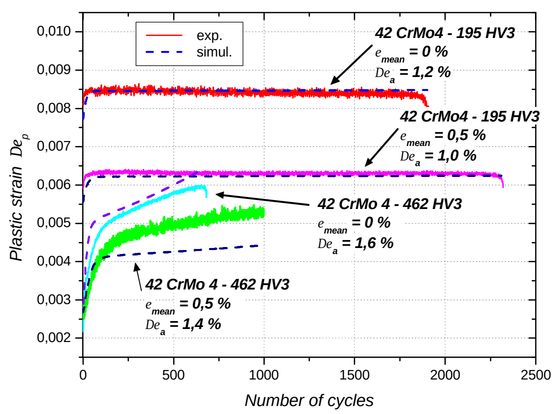

Figure 3. A comparison of numerical and experimental plastic strain ? e^{p}

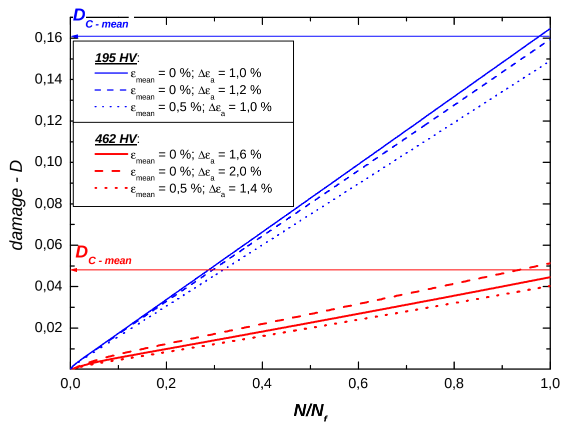

Figure 4. Numerical damage growth up to experimentally determined number of cycles N_f

4.2 Contact of the raceway and rolling element

The computational model has been applied to a test case related to large rolling bearings. Bearing rings are made of normalized steel 42 CrMo 4 with a hardness of 205 HV. The rolling surface was inductive hardened to the hardness of 630 HV and the depth of 0.6 mm. Different load cycle compression forces have been evaluated by the finite element simulation. Figure 5 shows the damage distribution in the

raceway after 100 load cycles and the material damage value through raceway depth at different load cycles where we can see the damage growth as a function of the number of loading cycles. The softening of the core material of the raceway in the process of repeated contact force is shown in Figure 6, which shows the stress and strain path as a function of load cycles. Figure 7 shows the stress-strain hysteresis loops as a function of the number of cycles at the most critical location in the raceway. The most critical location is the one at the location of the maximum damage value that occurs in the core material at the limit with the hardened layer.
The verification of the established numerical material model for use in determining the actual carrying capacity of the rolling contact in low speed axial bearings is made with the good comparison of experimental and numerical results of total and permanent raceway deformation growth in relation to the number of load cycles (Figure 8).

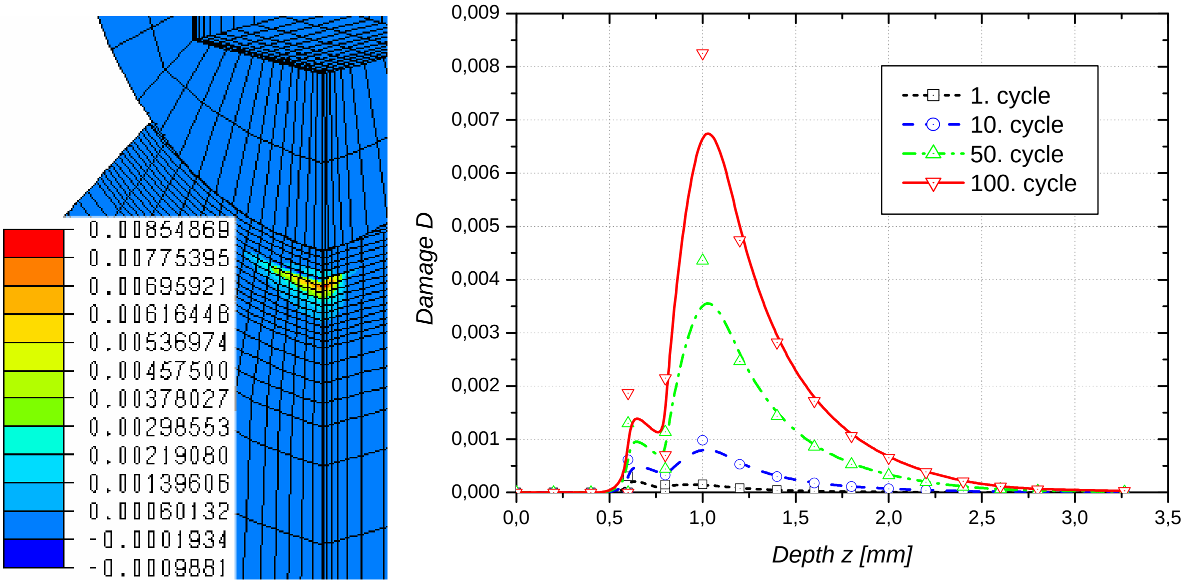

Figure 5. Comparison between measurement and numerical calculation for the tempered steel 42 CrMo 4 with a hardness of 462 HV. A non-symmetrical load case at a constant extension of ? e = 1.4 % and a mean value of e_{mean} = 0.5 %

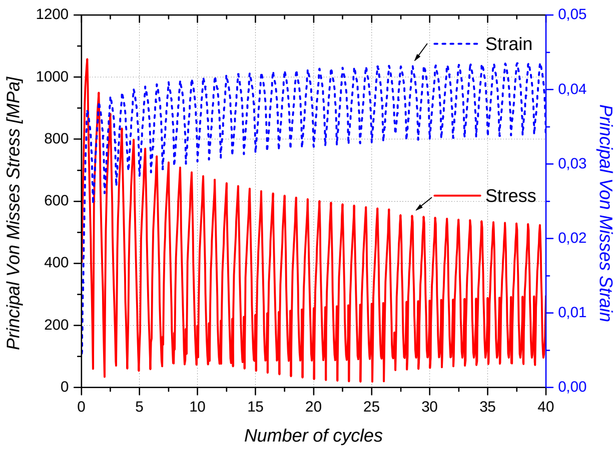

Figure 6. Principal Von Misses stress and strain in relation to the number of load cycles

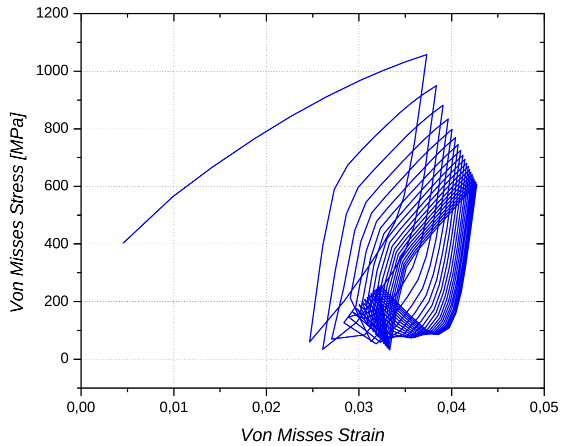

Figure 7. Von Misses strain-stress hysteresis loops in relation to the number of load cycles

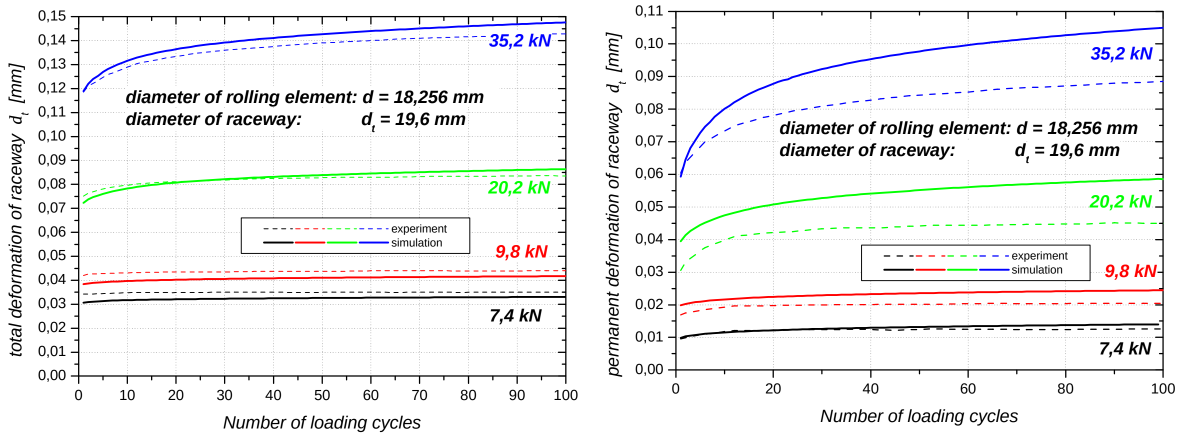

Figure 8. Comparison of experimental and numerical results of total and permanent raceway deformation growth in relation to the number of load cycles

5. Conclusions

A numerical model for analysis of low cycle carrying capacity for large bearings raceway under cyclic loading has been presented. The model combines isotropic and kinematic hardening or softening with continuum damage mechanics. Preliminary analyses show that stress-strain situation in critical regions of raceway can change significantly with increasing number of forming cycles. This suggests that estimations of carrying capacity should be based on the analysis of cyclic plasticity and damage, which have a considerable effect on the service life and cause redistribution of stress-strain fields in the vicinity of critical regions.

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