10TH INTERNATIONAL DESIGN STRUCTURE MATRIX CONFERENCE, DSM'08
11 – 12 NOVEMBER 2008, STOCKHOLM, SWEDEN

# CYCLES IN THE MULTIPLE-DOMAIN MATRIX – INTERPRETATION AND APPLICATIONS

**Wieland Biedermann and Udo Lindemann**
Chair for Product Development, Technische Universität München

*Keywords: DSM, MDM, graph theory, structural analyses, deduction of indirect dependencies*

## 1 INTRODUCTION

The design structure matrix (DSM) [1] and the domain mapping matrix (DMM) [2] are well established methods for dealing with complex systems and situations. However, they only allow for a limited view on a certain problem. To manage even more complex system the multiple-domain matrix (MDM) which comprises DSM and DMM methods was developed [3].
The MDM represents the basic structure of a complex system and defines the problem domains and their relations. The MDM structure can be modeled as a DSM like any other structure of relations. Thus the MDM itself forms a very abstract form of a DSM but so far no analysis of the MDM with DSM methods has been discussed.
In this paper we discuss the structural properties of DSMs and MDMs and show that DSM methods can be applied to the MDM. First, the structural properties of the DSM, the DMM, and the MDM are compared. Next we apply the cycle analysis of the DSM to the MDM. After that we will show that cycles can interpreted as formulas which compute new DSMs. Finally, we verify our results with an example taken from [4].

## 2 STRUCTURAL PROPERTIES OF THE MULTIPLE-DOMAIN MATRIX

There are three main properties to distinguish structure types. First, there are reflexive relations which link one element to itself. Then there are multiple relations between two elements. Lastly, there is the directedness of the relations which allows the distinction of the order of the related nodes. Additional restrictions of a structure can be defined for special analysis purposes. Based on these criteria the main classes of structures can be defined [5].

*Table 1. Structural properties of the DSM, the DMM and the MDM*

| properties | DSM | DMM | MDM |
|---|---|---|---|
| reflexive relations | no | no | yes |
| multiple relations between two elements | no | no | yes |
| additional restrictions | no | yes | no |
| directed relations | yes/no | no | no |
| resulting structure typ (graph theory) | digraph (directed) or simple graph (undirected) | bipartite graph | multigraph |

Both DSMs and DMMs do not allow for reflexive or multiple relations in their standard forms ([1], [2]), whereas the MDM comprises both [3]. A DSM can be directed or undirected. A DMM is normally undirected as it only represents a mapping between two domains which works in both directions. As the DMM is undirected the MDM is undirected as well. The non-reflexive relations within the MDM are DMMs and therefore undirected. The DMM has to fulfil an additional restriction. Its axes may not overlap i.e. the axes must not have elements in common.

## 3 APPLICATION AND INTERPRETATION OF THE CYCLE ANALYSIS

There are many different analyses in the DSM (see [3] for an overview). There are reordering methods like clustering, characterizations of single elements and relations like the active sum as well as criteria like cycles and paths. A lot of these methods can be applied to the MDM. In this paper we will only discuss cycles.
The relations in the MDM represent matrices like DSMs and DMMs. Thus the cycles in the MDM represent sequences of matrices. These define formulas for the computation of new matrices. Cycles define formulas which compute DSMs.

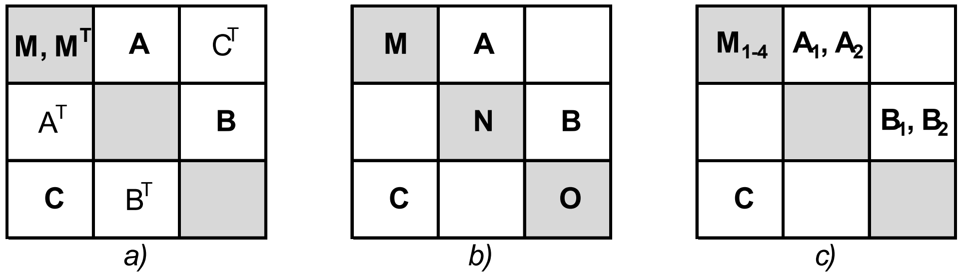



*Figure 1. Variations of computation defined by a single cycle: a) variation of the cycle direction, b) variation of the target domain, c) variation of the applied matrices*

One cycle defines a multitude of formulas. Firstly, there are two different possibilities to go through a cycle: clockwise and counter-clockwise. Therefore, a single cycle within a MDM defines two different computations. The results differ only in the directions of the relations and do not have different interpretations. When changing the direction the matrices have to be transposed to. As the order of the matrices is inverted and the matrices are transposed, the results are transposes of one another as shown by formula (1). Each cycle comprises different domains. The cycle starts and ends at each domain. Therefore, it is possible to derive a formula to compute a DSM for each domain. Formula (2) gives examples for such formulas (see figure 1b for a definition of the symbols). As shown, each cycle defines a formula for each domain it comprises. The computations can be varied further if multiple DMMs link two domains. Each DMM combination defines a unique computation. Formula (3) gives examples for such formulas (see figure 1c for a definition of the symbols).

$$M=A\cdot B\cdot C \qquad M^T=(A\cdot B\cdot C)^T=C^T\cdot B^T\cdot A^T \tag{1}$$

$$M=A\cdot B\cdot C \qquad N=B\cdot C\cdot A \qquad O=C\cdot A\cdot B \tag{2}$$

$$M_1=A_1\cdot B_1\cdot C \qquad M_2=A_1\cdot B_2\cdot C \qquad M_3=A_2\cdot B_1\cdot C \qquad M_4=A_2\cdot B_2\cdot C \tag{3}$$

A single cycle describes a multitude of computations and most undirected graphs comprise many cycles. Therefore, it is quite a hard task to choose those computations, which result in an appropriate DSM for the analysis purpose. Maurer developed a method to compute a simple interpretation for networks deduced in the MDM [3]. Each native DMM or DSM gets a simple interpretation and forms a building block. These interpretations can then be linked along the matrices in the formula to form the result’s interpretation. The same method can be applied to compute the interpretation of the results of the cycle formulas. The next chapter gives an example for the application of the method.

# 4 CASE STUDY – DEVELOPMENT OF A RACECAR

The example MDM describes the development of a racecar. The racecar is a student project (TUfast) at the Technische Universität München and is developed for the formula student. The MDM comprises all structural information relevant for the development of the racecar from TUfast’s point of view. The MDM contains six domains: Requirements, persons, data/documents, parts, process steps and milestones. The domains are linked by 13 DMMs and two domains include a DSM [5].

*Table 2. Cycles in the example MDM (extract)*

| No. | Domain 1 | Domain 2 | Domain 3 | Domain 4 | Domain 5 | Domain 6 | Variations | Domains | Total |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Part | Data/document | | | | | 1 | 2 | 2 |
| 2 | Person | Data/document | | | | | 1 | 2 | 2 |
| 3 | Data/document | Process step | | | | | 1 | 2 | 2 |
| 4 | Part | Person | Requirement | | | | 1 | 3 | 3 |
| 5 | Part | Person | Data/document | | | | 4 | 3 | 12 |
| 6 | Part | Data/document | Milestone | | | | 2 | 3 | 6 |
| … | … | … | … | … | … | … | … | … | … |
| 24 | Requirement | Part | Milestone | Process step | Data/document | Person | 4 | 6 | 24 |
| | | | | | | | 60 | 98 | 256 |

The MDM was transformed to a multigraph. Then, the cycles formed by the DMMs were computed. The relations form 24 basic cycles (see table 2 for an extract). The cycles describe 256 computations. The different directions of the cycles were not taken into account, as the neither result in different matrices nor change the resulting interpretation.

*Table 3. Interpretation of different variants of one cycle*

| No. | Domain 1 | DMM I | Domain 2 | DMM II | Domain 3 | DMM III | Domain 4 |
|---|---|---|---|---|---|---|---|
| 5a | Part A | designed by | Person B | generates | Data/ Document C | represents | Part D |
| 5b | | | | generates | | required by | |
| 5c | | | | needs | | represents | |
| 5d | | | | needs | | required by | |

To be able to choose the right computations for the system analysis the rough interpretations of the cycles were determined. Table 3 shows some of the different interpretations of cycle 5. The interpretation can be further detailed and better formulated, e.g. the interpretation of 5c would read "Two parts are linked, if the first part is designed by a person, who needs a document, which represents the second part." Using the interpretations formulas were chosen which resulted in new DSMs. These DSMs allowed additional analyses and insights into the racecar's developments.

## 5 CONCLUSION AND OUTLOOK

The MDM is a powerful tool for analyzing and managing complex situations and problems. Though its basic characteristics have been stated, it is hardly investigated. In this paper two major ideas were discussed. First the application of DSM analyses to the MDM and second the systematic determination of formulas for the deduction of indirect dependencies. Both ideas contribute to a deeper understanding of the MDM and allow for a more systematic analysis of complex systems. In future research the deduction of indirect dependencies has to be discussed in more detail and the appropriate formulas have to be classified. Another field of research is the application of DSM, DMM and other structural analyses to the MDM. In this paper the first steps and their promising results were shown.

## ACKNOWLEDGEMENTS

We thank the Deutsche Forschungsgemeinschaft (DFG) for funding this research as part of the collaborative research centre "Managing cycles in innovation processes – Integrated development of product service systems based on technical products" (SFB 768).

## REFERENCES

[1] Browning T. R. Applying the Design Structure Matrix to System Decomposition and Integration Problems: A Review and New Directions. In *IEEE Transactions on Engineering Management, 48(3)*, 2001, pp. 292-306

[2] Danilovic M. and Browning T. R. A Formal Approach for Domain Mapping Matrices (DMM) to Complement Design Structure Matrices (DSM). In *Proceedings of the 6th Design Structure Matrix (DSM) International Workshop*, Cambridge, 2004 (University of Cambridge, Engineering Design Centre)

[3] Maurer M. *Structural Awareness in Complex Product Design*, 2007 (Dr. Hut, München)

[4] Maurer M. and Lindemann U. Facing Multi-Domain Complexity in Product Development. In *Cidad Working Paper Series 3*, 2007(1), 2007, pp. 1-12

[5] Gross J. L. and Yellen J. *Graph Theory and its Applications.* 2nd Ed., 2006 (Chapman & Hall/CRC, Boca Raton)

Contact: Wieland Biedermann
Technische Universität München
Chair for Product Development
Boltzmannstr. 15
85748 Garching
Germany
+49 (89) 28 91 51 - 35
wieland.biedermann@pe.mw.tum.de
http://www.pe.mw.tum.de/

10TH INTERNATIONAL DSM CONFERENCE

# Cycles in the Multiple-Domain Matrix Interpretation and Applications

Wieland Biedermann and Udo Lindemann

Institude for Product Development
Technische Universität München

Technische Universität München



## Agenda

- Motivation
- The Multiple-Domain Matrix (MDM)
- Structural properties of the DSM, the DMM, and the MDM
- Cycles in the MDM
- Interpretation of cycles in the MDM as computations
- Variations of computations
- Example: Development of a racecar
- Application of cycles to the example
- Conclusion and outlook

Technische Universität München

## Motivation

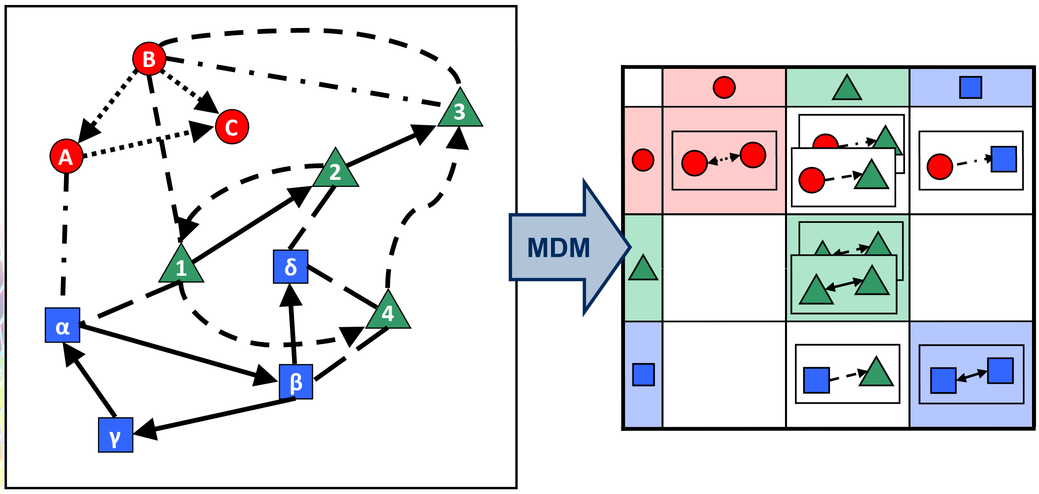
- The multiple-domain matrix (MDM) combines design structure matrices (DSM) and domain mapping matrices (DMM)
- The MDM has been introduced to enhance the possibilities of the DSM
- The MDM forms a structure which seems to be similar to the DSM
- There are manifold analyses for the DSM

→ Can DSM analyses be applied to the MDM?
→ How can the results be interpreted?





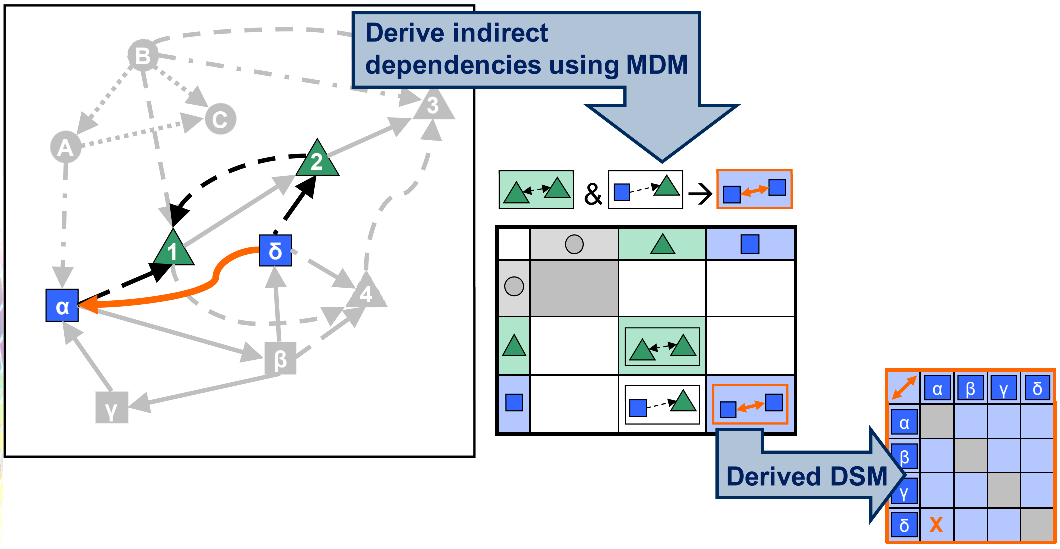
## The Multiple-Domain Matrix (MDM)




→ Based on native DMMs and DSMs new matric can be computed

Based on [Maurer 2007]

## Deriving indirect dependencies



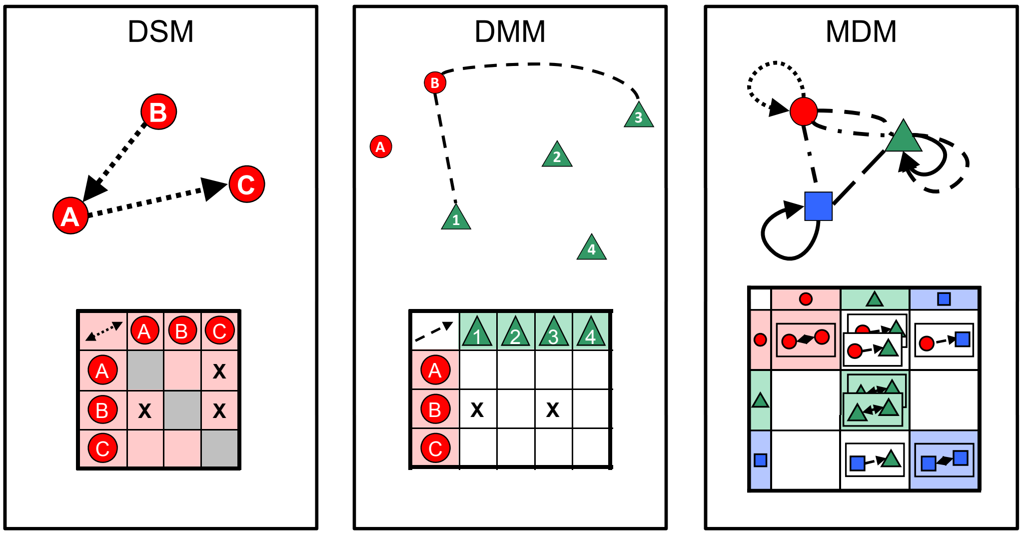

→ Based on native DMMs and DSMs new matric can be computed

Based on [Maurer 2007]





## Structural Properties of the DSM, the DMM and the MDM




→ Though some differences exist DSM analyses can be applied to the MDM

Based on [Maurer 2007]

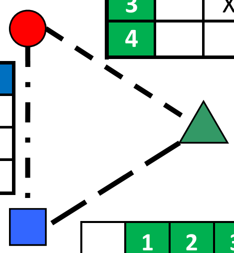
## Cycles in the MDM and their interpretation

DMM 3

| | A | B | C |
|---|---|---|---|
| 1 | | X | |
| 2 | | | |
| 3 | | X | |
| 4 | | | |

DMM 1

| | α | β | γ | δ |
|---|---|---|---|---|
| A | X | | | |
| B | | | | X |
| C | | | | |



| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| α | X | | | |
| β | | | | X |
| γ | | | | |
| δ | | X | | X |

DMM 2

- A cycle is a closed sequence of edges in a graph
- The (non-reflexive) edges in the MDM represent DMMs

→ A cycle in the MDM represents a sequence of DMMs

- DMMs are matrices
- Matrix multiplications are used to derive new DSMs in the MDM

→ Cycles in the MDM define formulas to compute new DSMs

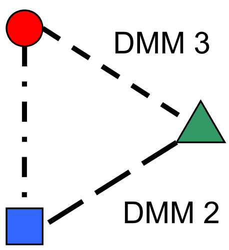
DSM = DMM 1 · DMM 2 · DMM 3

→There are more possible computations than those described in [Maurer 2007]





## Variation of the computation – clockwise and counter-clockwise




| | A | B | C |
|---|---|---|---|
| A | | X | |
| B | | | |
| C | | | |

DSM 1 = DMM 1 · DMM 2 · DMM 3

| | A | B | C |
|---|---|---|---|
| A | | | |
| B | X | | |
| C | | | |

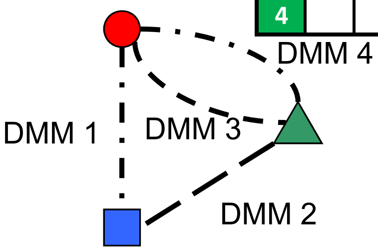
DSM 2 = DMM $3^T$ · DMM $2^T$ · DMM $1^T$

- You can define two different cycles: clockwise and counter-clockwise.
- Each cycle describes an own computation
- The resulting DSMs are transposes of each other

→Both computations are equivalent and need no distinction

## Variation of the computation – multiple DMMs

| | A | B | C |
|---|---|---|---|
| 1 | | | X |
| 2 | | | |
| 3 | | X | |
| 4 | | | |




| | A | B | C |
|---|---|---|---|
| A | | X | |
| B | | | |
| C | | | |

DSM 1 = DMM 1 · DMM 2 · DMM 3

| | A | B | C |
|---|---|---|---|
| A | | | X |
| B | | | |
| C | | | |

DSM 2 = DMM 1 · DMM 2 · DMM 4

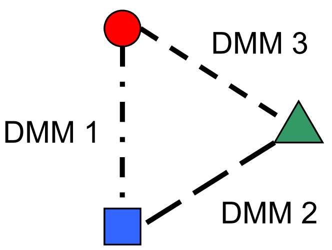
- Each DMM combination defines an own computation.
- Each computation results in different DSMs

→The computations are not equivalent
→The resulting DSMs must be analyzed separately





## Variation of the computation – multiple starting points




| | A | B | C |
|---|---|---|---|
| A | | X | |
| B | | | |
| C | | | |

DSM 1 = DMM 1 · DMM 2 · DMM 3

| | α | β | γ | δ |
|---|---|---|---|---|
| α | | | | X |
| β | | | | |
| γ | | | | |
| δ | | | | |

DSM 2 = DMM 2 · DMM 3 · DMM 1

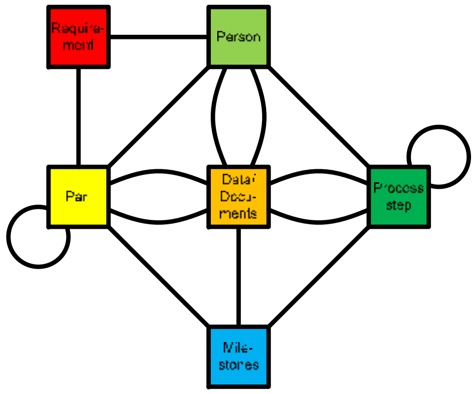
- Each cycle can start and end at each domain it comprises

→Each cycle defines a computation for a DSM in each domain it comprises

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | | X | | X |
| 2 | | | | |
| 3 | | X | | X |
| 4 | | | | |

DSM 3 = DMM 3 · DMM 1 · DMM 2

## Example development of a racecar




**TU*fast***
- Student group (TUM) developing a racecar
- National and international competitions

→ Strict cost and safety requirements
→ Extensive documentation

**Analysis of the racecar development**
- The MDM comprises all aspects relevant for the development
- The MDM consists of 2 native DSMs and 13 native DMMs



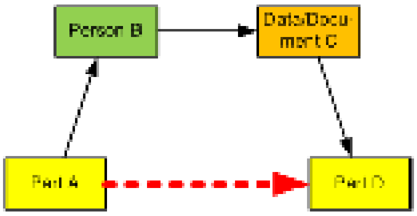


## The cycles in the MDM

| No. | Domain 1 | Domain 2 | Domain 3 | Domain 4 | Domain 5 | Domain 6 | Variations | Domains | Total |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Part | Data/document | | | | | 1 | 2 | 2 |
| 2 | Person | Data/document | | | | | 1 | 2 | 2 |
| 3 | Data/document | Process step | | | | | 1 | 2 | 2 |
| 4 | Part | Person | Requirement | | | | 1 | 3 | 3 |
| 5 | Part | Person | Data/document | | | | 4 | 3 | 12 |
| 6 | Part | Data/document | Milestone | | | | 2 | 3 | 6 |
| 7 | Person | Data/document | Process step | | | | 4 | 3 | 12 |
| 8 | Data/document | Process step | Milestone | | | | 2 | 3 | 6 |
| 9 | Requirement | Part | Data/document | Person | | | 4 | 4 | 16 |
| 10 | Part | Person | Data/document | Milestone | | | 2 | 4 | 8 |
| 11 | Part | Person | Process step | Data/document | | | 4 | 4 | 16 |
| 12 | Part | Person | Process step | Milestone | | | 1 | 4 | 4 |
| 13 | Part | Data/document | Process step | Milestone | | | 4 | 4 | 16 |
| 14 | Person | Data/document | Milestone | Process step | | | 2 | 4 | 8 |
| 15 | Requirement | Part | Milestone | Data/document | Person | | 2 | 5 | 10 |
| 16 | Requirement | Part | Milestone | Process step | Person | | 1 | 5 | 5 |
| 17 | Requirement | Part | Data/document | Process step | Person | | 4 | 5 | 20 |
| 18 | Part | Person | Data/document | Process step | Milestone | | 4 | 5 | 20 |
| 19 | Part | Person | Process step | Data/document | Milestone | | 2 | 5 | 10 |
| 20 | Part | Person | Process step | Milestone | Data/document | | 2 | 5 | 10 |
| 21 | Part | Data/document | Person | Process step | Milestone | | 4 | 5 | 20 |
| 22 | Requirement | Part | Data/document | Milestone | Process step | Person | 2 | 6 | 12 |
| 23 | Requirement | Part | Milestone | Data/document | Process step | Person | 2 | 6 | 12 |
| 24 | Requirement | Part | Milestone | Process step | Data/document | Person | 4 | 6 | 24 |
| | | | | | | | 60 | 98 | 256 |

**→ How to choose the right computation for a specific purpose?**

## Interpretation of the cycles




- Each DMM has a specific interpretation
- The interpretation of a computed DSM can be derived from the DMM interpretations
- Derived by linking the DMM interpretations

| No. | Domain 1 | DMM I | Domain 2 | DMM II | Domain 3 | DMM III | Domain 4 |
|---|---|---|---|---|---|---|---|
| 5a | Part A | designed by | Person B | generates | Data/ Document C | represents | Part D |
| 5b | | | | generates | | required by | |
| 5c | | | | needs | | represents | |
| 5d | | | | needs | | required by | |

**Interpretation of Cycles 5c:**

"Two parts are linked, if the first part is designed by a person, who needs a document, which represents the second part."

→ Interpretation for the result of each computation can be computed

→ Based on the interpretation those computations can be chosen which help the analyses purpose





## Conclusion and Outlook

- MDM has similar but not the same structural properties as the DSM

→ DSM analyses can be applied to the MDM

- Cycles in the MDM describe computations

→ Formulas for deriving indirect dependencies can be determined systematically.

- Future research will address both ideas in more detail

→ Application of other DSM or DMM analyses to the MDM

→ Discussion, systematization and classification of formulas for deriving indirect dependencies