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Describing a mechanical model of a novel axial piston type steam expander

Rasmus Grip, Jan-Gunnar Persson
System and component design
Brinellvägen 83
114 28 Stockholm
SWEDEN
grip@md.kth.se ***jgp@md.kth.se***

Abstract
A conceptual design describing an axial piston type steam expander has been proposed by the Swedish company Ranotor. Previously its kinematics has partly been described by O. R. Lang [1] and fully in an article by the main author of this paper [2].

This expander concept differs from a conventional piston engine in the aspects that the pistons are axial-symmetrically placed around the engine shaft and act on this via a so called wobble plate. The working medium, over-critical steam will perform the actual net shaft work in a closed thermal cycle by expanding the steam in the cylinders via the translation of the pistons. Other than within a steam cycle, the expander could also be suitable for applications such as AC-compressors, ORC-cycles etc.

A kinematical and constraint dynamical rigid body mechanics model has been developed. The method is that of analytical vector analysis method, which have been performed in the general symbolic representation software Maple. The output from this model is position and rotational vectors with derivatives and internal and external forces and torques. Parameter values such as component mass, mass moment of inertia and geometry as well as steam data and operational parameters are the input of the model.

The output of the model supply a foundation where one can study the forces and velocities in the joints in the design phase, and further serve as a basis for tribology rig testing of steam/water lubricated sliding contacts in bearings and piston/cylinder. The overall goal of this project is to construct a functional prototype of this expander. A brief presentation of the lateral piston force vector for a set of operational cases is presented and discussed. The external and internal forces are presented as graphs generated in the software Matlab and the sliding velocity as an equation in the output example chapter.

Keywords: Constraint mechanics, vector analysis, joint interfaces, multi-body, mechanical work.

1 Introduction

The main objective of the project related to this article is to construct a functional prototype of the steam expander (Figure 1) presented in the context of the RAN-concept [3].

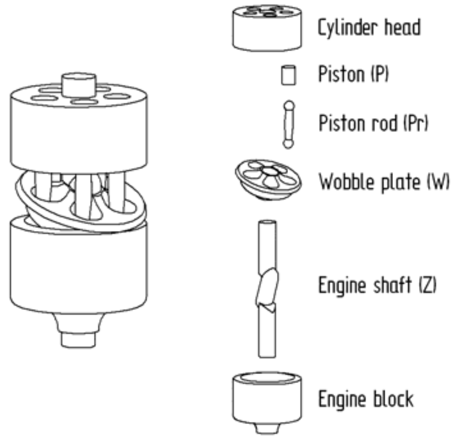

Figure 1. A schematic illustration of the steam expander presented in the RAN-concept.

To aid in the design and construction of such a prototype, we have chosen to build a kinematical and a constrained dynamical model of the expander. The argument behind this decision is that such a model produces results that make predictions of the forces, torques and relative velocities in the joints, i.e. the contacts interfaces between the ingoing parts of the expander, when varying construction parameters and operational cases. These predictions serve a basis for an understanding of the internal force and torque distribution, demands on joint designs and further in the preparation of tribologic testing where two or more parts are subjected to contact at a relative velocity and subjected to some contact force.

A review of the kinematical and dynamical model is presented in the “method” chapter. Since the motion of the expander is quite complex for one to visualize; a series of snapshots over one engine shaft revolution is presented below (Figure 2):

Figure 2. The motion of the expander illustrated in a series of snapshots over one engine shaft revolution.

2 Method

As described in the introductory chapter, the method used in the work behind this article is a mechanical rigid body constraint analysis, described in two parts below; first a kinematical model of the expander, then the constraint model.

2.1 Kinematical model

The kinematical model of the expander has previously been described in an article by the main author of this paper [2], what follows is a brief recapitulation. The kinematical model describes the translation and rotation of the ingoing parts of the mechanism, i.e. positioning and rotational vectors. Body fixed coordinate systems are defined and thus rotational matrices (matrices governing the transformation from one coordinate system to another) can be created for each ingoing part of the expander (Figure 3).

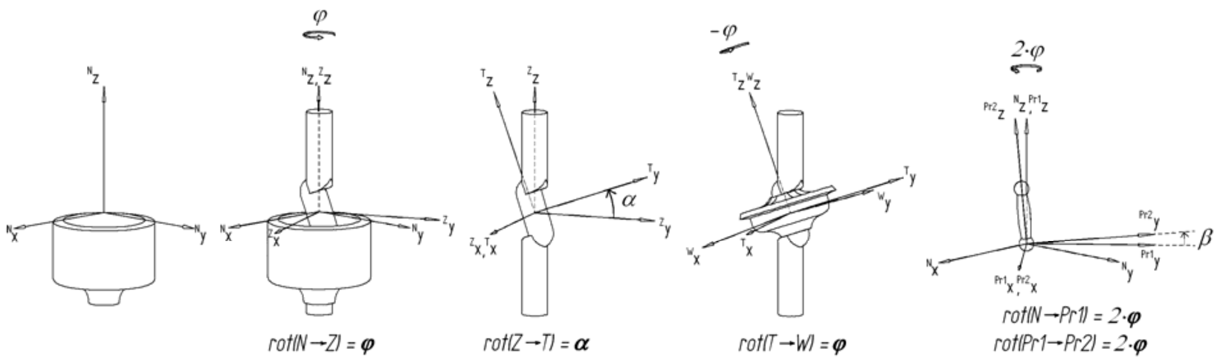

Figure 3. From left to right: Space fixed coordinate system (N), engine shaft system (Z), tilted system (T), wobble plate system (W), piston rod system 1 (Pr1) and 2 (Pr2).

The coordinate systems are related to each other via rotations about common axes. Since body fixed coordinate systems are available, vectors describing points in one body can be written in each body fixed system and transformed to any other arbitrary system, following:

$$ {}^{A}\left[\bar{r}_P\right] = \left[{}^{B\rightarrow A}R\right] \bullet {}^{B}\left[\bar{r}_P\right] \quad (1) $$

Where $\left[{}^{B\rightarrow A}R\right]$ is the rotational matrix describing the transformation a vector. Here; the vector $\bar{r}_P$ undergoes a transformation from the B-system (${}^{B}\left[\bar{r}_P\right]$) to the A-system (${}^{A}\left[\bar{r}_P\right]$).

The relevant parameters when defining the geometry is illustrated below (Figure 4). TDC is short for Top Dead Centre and BDC for Bottom Dead Centre, although this kind of expander does not experience any “dead centre” since the force vector from the steam pressure inside the cylinder never coincides with the piston rod’s axis.

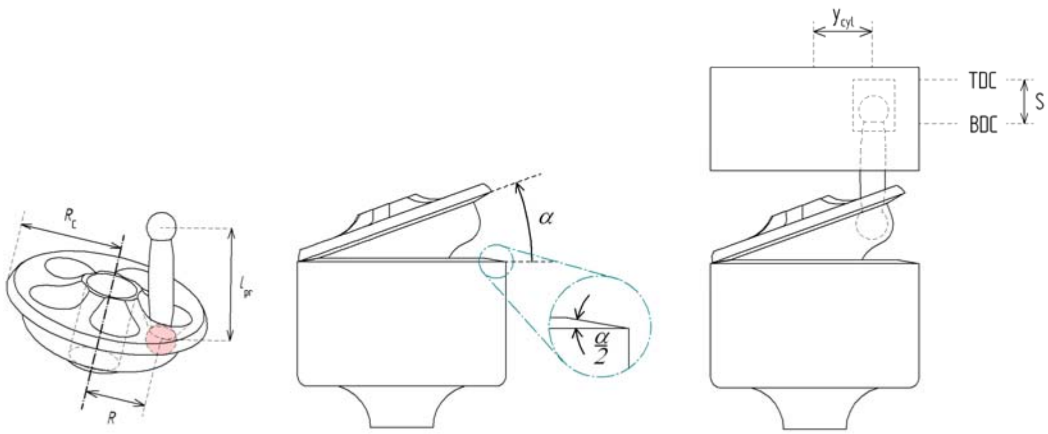

Figure 4. The five parameters R_c, R, l_{pr}, α , S and one geometrical condition (y_{cyl}) needed in order to define the geometry of the model.

Results from this analysis shows that the piston translates in a completely harmonic fashion, and that the piston rods have a constant angle of deflection, given the geometrical condition:

$$y_{cyl} = \frac{1}{2} \cdot R \cdot (1 + \cos(\alpha)) \quad (2)$$

2.2 Constraint formulation

All ingoing parts of the expander are modeled as rigid with a clearly defined center of mass vector e.g. $\bar{r}_G$, and rotational orientation e.g. $\bar{\varphi}$. In 3D-space such bodies are said to exhibit six degrees of freedom; i.e. the center of mass vector is described by three linear independent translations and the rotational orientation is described by three linear independent rotations. The relation between force and acceleration and torque and rotation is described by the equations:

$$\bar{F} = m \cdot \ddot{\bar{r}}_G \quad (3)$$

$$\bar{M} + \bar{r}_F \times \bar{F} = J \cdot \ddot{\bar{\varphi}} \quad (4)$$

The focus in this paper is to study forces and torques when the system of rigid bodies undergoes a constrained motion. The translational acceleration $\ddot{\bar{r}}_G$ and rotational acceleration $\ddot{\bar{\varphi}}$ are thus known, and we must consider the force and torques as unknown. These unknown forces and torques are named $\bar{R}$ and $\bar{T}$. Applied known forces and torques are named $\bar{F}_a$ and $\bar{M}_a$. The force equation and torque equation is thereby formulated:

$$\bar{R} + \bar{F}_a = m \cdot \ddot{\bar{r}}_G \quad (3')$$

$$\bar{T} + \bar{M}_a + \bar{r}_R \times \bar{R} + \bar{r}_F \times \bar{F}_a = J \cdot \ddot{\bar{\varphi}} \quad (4')$$

Where $\bar{r}_R$ and $\bar{r}_F$ are the vectors between the centre of mass G and the point of attack of $\bar{R}$ and $\bar{F}_a$ respectively. The picture (Figure 5) below illustrates what is said above:

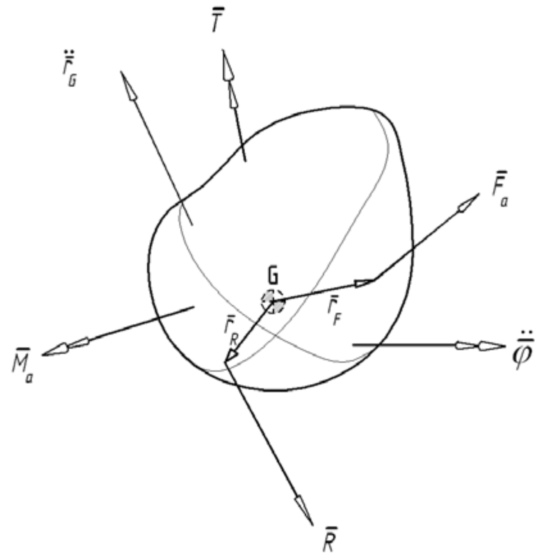

Figure 5. The figure depicts a rigid body with constraint and applied forces and torques, translational acceleration and rotational acceleration.

The aim of the work behind this article is to obtain the forces and torques in the joints of the expander. A body or system of bodies can be cut at an arbitrary point or a part can be seen as decoupled from the original body or system. If internal forces and torques are introduced in this cut or between the system and the decoupled part one can calculate the forces and torques demanded in order to constrain the part to the body or to the system. On a system level, i.e. here; the whole of the expander, the input of work shall equal the output work. Equations (3') and (4') imply that the constraint forces and torques can perform work. In order to avoid performing work on the system where such shall not occur, the constraint forces and torques are introduced as couples of opposite directed forces or torques:

$$W_R = \int_{\bar{s}_1}^{\bar{s}_2} \overline{F} \bullet d\bar{s} = \int_{\bar{s}_1}^{\bar{s}_2} \left(\overline{R} - \overline{R}\right) \bullet d\bar{s} = 0 \tag{5}$$

$$W_T = \int_{\bar{s}_1}^{\bar{s}_2} \overline{M} \bullet d\overline{\varphi} = \int_{\bar{s}_1}^{\bar{s}_2} \left(\overline{T} - \overline{T}\right) \bullet d\overline{\varphi} = 0 \tag{6}$$

A constraint force or torque can also be introduced without a counter-directed constraint and not perform work if there is no instantaneous relative motion at the point of attack (i.e. $d\bar{s} = \overline{0}$, $d\overline{\varphi} = \overline{0}$), as for example in a no-slip situation like between the wobble plate and the crank case. In our model, the only constraint that shall perform work is the torque constraining the engine shaft to the prescribed rotation; this could be seen as an ideal engine brake.

The expander consists of five pistons and five piston rods, one wobble plate and one engine shaft. These are theoretically coupled to each other via joints; ball joints between piston and piston rod, ball joints between piston rods and wobble plate and a revolute joint between wobble plate and engine shaft. The pistons are coupled to ground (i.e. the cylinder head) via translational joints, the wobble plate is coupled to the crank case via a gear joint and the engine shaft is coupled to ground (i.e. crank case) via a revolute joint. The relevant forces and torques together with the joints are illustrated below (Figure 6).

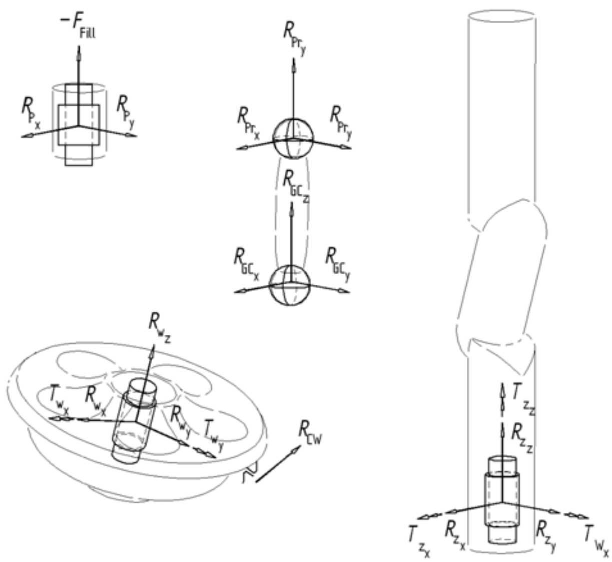

Figure 6. The relevant constraint forces and torques on the ingoing parts of the expander.

3 Verifying the model

In order to test the feasibility of the output from the model, it is subjected to some tests.

3.1 Input and output of mechanical work

First, the amount of mechanical work input shall equal the output, i.e. according:

$$W_{input} = \int_0^{2\cdot\pi} p(\varphi)\cdot V(\varphi) d\varphi = W_{output} - 2\cdot\pi\cdot\int_0^{2\cdot\pi} T_{z_Z}\cdot d\varphi \qquad (6')$$

Here, the steam pressure performs work, and the constraint torque on the engine shaft performs the counter work needed to keep the rotational velocity of the engine shaft to the specified value. An analytical verification of the equation (6') is by our means practically out of reach since the expression for the constraint torque T_{z_Z} is too complicated. But numerical evaluations show a converging agreement between the two sides of the equation when increasing the number of time-steps. Further, when setting the steam pressure to any constant value, no net input work is done to the expander and thus no constraint torque is needed to constraint the engine shaft revolution to a constant velocity – which the output from the model also shows.

3.2 Total force sum

Following equation (3'), the total sum of constraint and applied forces over the boundary of a system should equal the centre of mass acceleration times the total mass. If the whole of the expander is taken in concern, then the external constraint forces are those at the piston, engine shaft and wobble plate gear forces and the applied forces are the steam forces and gravity.

The trivial case where there are no steam forces and everything is at a stand still; the only external vertical force is the one constraining the engine shaft from translating. Thus, this force should equal the total weight of the system, which it does. The centre of mass acceleration is of course dependent on geometry parameters. If the wobble plate and engine shaft centre of masses coincide, then the expander is completely balances mass-wise since pistons and piston rods translate in a harmonic and counter interference fashion. Thus, equation (3') is fulfilled if the left side of the equation equal zero, which it does.

4 Simulation parameters

There are four sets of parameter values needed to specify in order to simulate the mechanical behaviour of the expander:

- material properties (mass, mass moment of inertia)
- geometrical parameters (lengths, angles)
- steam parameters (admission and condenser pressure, degrees of filling and of condenser pressure)
- rotational velocity.

The purpose of the model is to be able to study the influence of different parameters. This work has only begun, but to be able to produce graphs of the model output one have to specify numerical values of the necessary parameters. The specific values used here are only examples.

4.1 Material properties

As being in the design phase of the expander, no exact material is yet specified but we have chosen to perform the simulations with a density of 7900 kg/m^3 for all parts. This paired with the shapes of the parts as in (Figure 1) the masses and mass moment of inertias are as follows:

$$m_p = 0.408kg,\ J_{p_{xx}} = 0.437\cdot 10^{-3}kg/m^2,\ J_{p_{yy}} = 0.437\cdot 10^{-3}kg/m^2,\ J_{p_{zz}} = 0.093\cdot 10^{-3}kg/m^2$$
$$m_{pr} = 0.465kg,\ J_{pr_{xx}} = 0.754\cdot 10^{-3}kg/m^2,\ J_{pr_{yy}} = 0.754\cdot 10^{-3}kg/m^2,\ J_{pr_{zz}} = 0.034\cdot 10^{-3}kg/m^2$$
$$m_w = 1.639kg,\ J_{w_{xx}} = 3\cdot 10^{-3}kg/m^2,\ J_{w_{yy}} = 3\cdot 10^{-3}kg/m^2,\ J_{w_{zz}} = 6\cdot 10^{-3}kg/m^2$$
$$m_z = 1.565kg,\ J_{z_{xx}} = 21\cdot 10^{-3}kg/m^2,\ J_{z_{yy}} = 21\cdot 10^{-3}kg/m^2,\ J_{z_{zz}} = 1\cdot 10^{-3}kg/m^2$$

4.2 Geometrical parameters

The geometrical parameter values needed to define the geometry of the expander (Figure 4) are specified as follows:

$$l_{pr} = 0.2\,m,\ \alpha = 20°,\ S = 0.04\,m,\ R = 0.0585\,m,\ R_C = \frac{3}{2}\cdot R$$

4.3 Operational parameters

There are two operational parameters, filling angle and rotational velocity. There are four minimum and maximum values of these two operational parameters specified. The filling angle parameter describes how big a portion of one engine shaft revolution the steam is let into the cylinder, this in turn partly determines the resulting engine shaft torque. The angle of filling φ_{Fill} can be varied between *15* º and *90* º of one engine shaft revolution. Analogous, there is an angle of condenser, i.e. a portion of one revolution of the engine shaft for which the pressure inside the cylinder is at the condenser pressure. This is kept constant and symmetrically extended round bottom dead centre by 18º. The function, where expansion and recompression is isentropic with $\kappa = 1.3$, describing the pressure inside the cylinder is schematically plotted below (Figure 7).

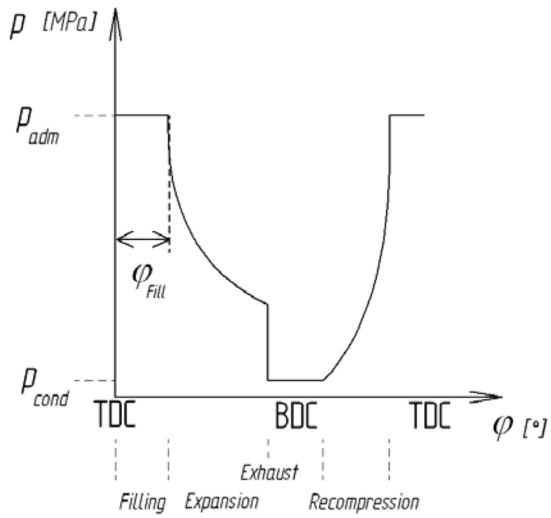

Figure 7. The pressure inside the cylinders is described by a piecewise continuous function as schematically plotted.

In this particular application, the pressures p_{adm}, p_{cond} and expansion and recompression are:

$$p_{adm} = 250 \cdot 10^5\,Pa,\ p_{cond} = 1 \cdot 10^5\,Pa,\ p_{isentropic}(\varphi) = p_0 \cdot \left(\frac{V(p_0)}{V(\varphi)}\right)^{\kappa},\ \kappa = 1.3$$

4.4 Rotational velocity

The angular velocity n range from *0* rpm to *6000* rpm. The simulations are run at constant velocity of revolution, thus the constraint torque on the engine shaft in the axial direction reflects the counter torque needed to keep a constant rotational velocity (analogous to a perfect engine brake).

5 Output example

One area of interest is the lateral constraint forces and the sliding velocity of the piston. Here, we plot the lateral piston force for three different rotational velocities *n = 2000* rpm, *4000* rpm and *6000* rpm without steam force (Figure 8); and for three different filling angles $\varphi_{Fill} = 15^o$, 45^o and 90^o at one rotational velocity *n = 2000* rpm (Figure 9).

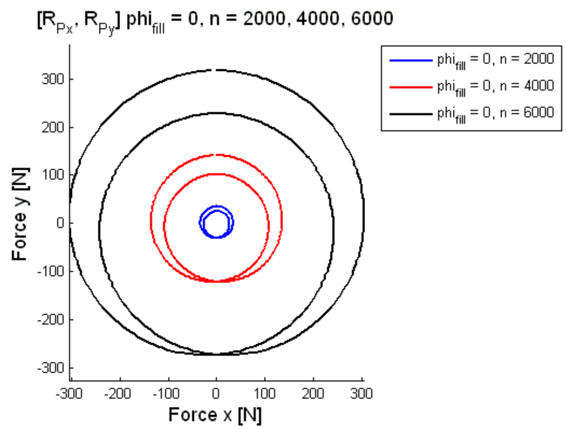

Figure 8. Lateral piston forces for three rotational velocities, *n = 2000* rpm, *n = 4000* rpm and *n = 6000* rpm. As seen, the lateral force increases as a function of *n*.

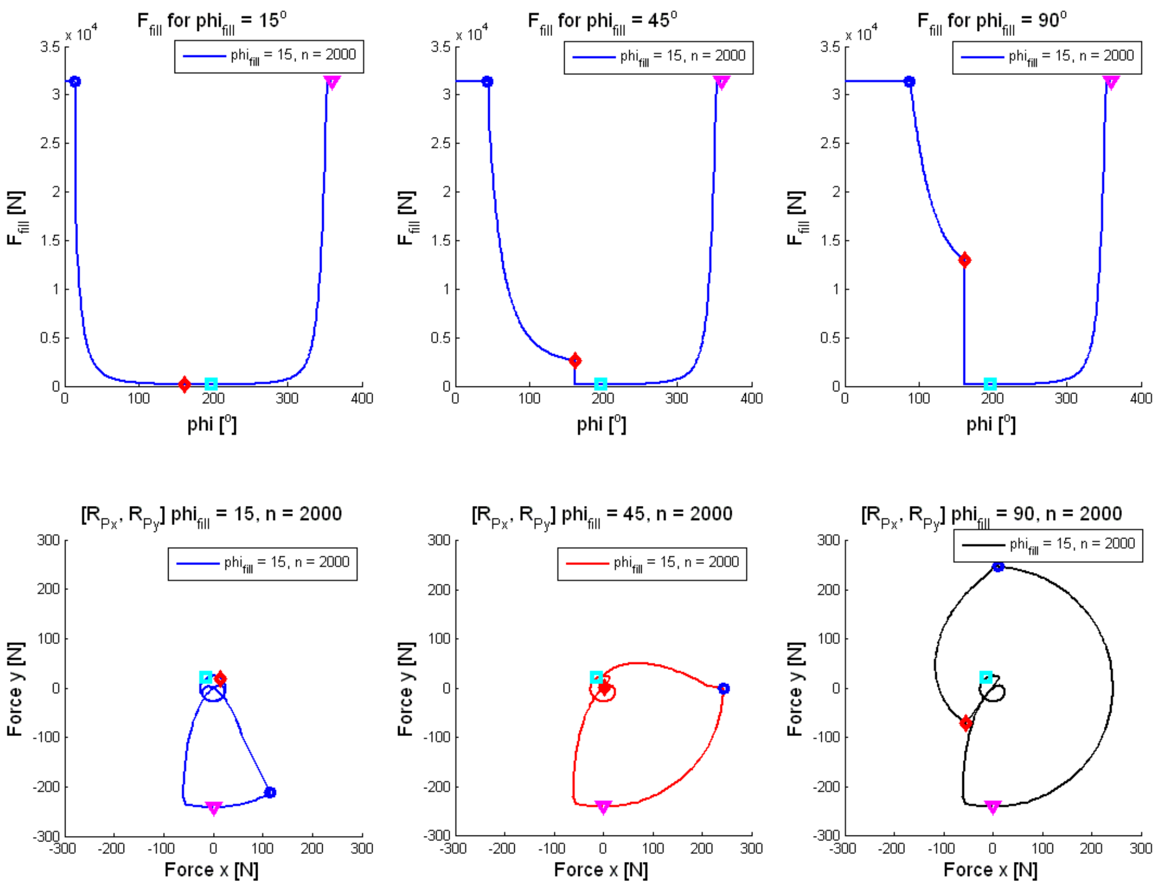

Figure 9. The upper row depicts the filling function for the steam force upon the piston. From left to right; $\varphi_{Fill} = 15^o$, $\varphi_{Fill} = 45^o$ and $\varphi_{Fill} = 90^o$ (see Figure 7). This is reflected in

the bottom row, which depicts the lateral piston force; the angle for the maximum force value elongates as φ_{Fill} increases. To aid the reading of the graphs, there are circles indicating end of filling, rhomb for steam outlet, square for start of recompression and diamonds for end of recompression. As seen, the innermost part of each graph in the bottom row contains part of the graph from $n = 2000$ rpm without steam forces.

The graphs can be understood by studying the illustration of the difference between mass forces and gas forces (Figure 10):

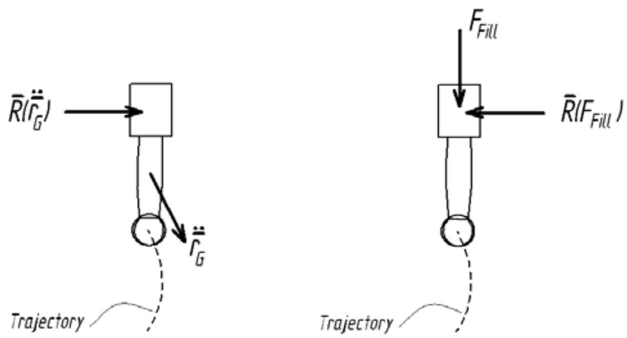

Figure 10. The constraint force originating from the centre of mass acceleration is (here momentarily) opposite from the force originating from the steam force. As seen in Figure 8 the constraint force vector makes two circles, and these are partly contained in the graphs where steam force is applied upon the piston (Figure 9).

The piston velocity is as mentioned sinusoidal, governed by the following equation (7):

$$\dot{r}_{G_P} = -R \cdot \sin(\alpha) \cdot \dot{\varphi} \cdot \sin(\varphi) \tag{7}$$

6 Conclusions

- A complete dynamical constraint model has been constructed and a method to construct such models has briefly been described.
- Expressions for the forces, torques and sliding velocities in the joint interfaces as a function of constructional parameters and operational cases have been derived.
- There is an ability to implement a sensitivity analysis and thus explore influence on the response of different parameters values and distributions.
- Construction parameters can be chosen such that easier joint designs can be realised, e.g. by weight optimization etc.
- Future tribological testing and detailed construction work is now one step closer.

This concludes the contents of this paper.

7 References

[1] O.R. Lang, “Triebwerke schnellaufender Verbrennungsmotoren”, Springer 1966, pp. 9-14.

[2] R. Grip, “Axial piston machine kinematics”, KTH 2007.

[3] “Rapport över RAN-projektet”, Nutek Diarienr 8522-93-12116