USE OF BISPECTRAL ANALYSIS IN PROGNOSIS OF THE SYSTEM DESTRUCTION PROCESS

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Abstract: *The goal of the paper is to develop a new prognostic strategy of system operation required for maintaining high level of readiness of complex power transmission systems. The most fundamental challenge is the fact that damage is typically a local phenomenon and may not significantly influence the lower-frequency global response of structures that is normally measured during vibration tests. Taking into account the physical aspect of phenomena under consideration, the special attention has been given to evaluation in terms of suitability of multidimensional spectra of a signal generated by the diagnosed kinematic pair. Next the bispectral analysis, a technique based on higher-order spectra is presented and applied to explore for different types of nonlinearity specially between quadratic or cubic one. The parametric interaction of that is illustrated.*

1. INTRODUCTION

In industrial reality is important to design products that adequately accomplishes desired functions with a minimum amount of failures. When failures analysis and prevention are coupled with the design process from its conception, diagnostics procedures should be set up according to the potential faults. Therefore in this paper, the possibillity of the preliminary study of requirements for the condition monitoring and diagnostics system of a machine is explored.

Research on applying model-supported vibroacoustic diagnosis in the examination and simulation of fatigue (wear and tear) and destruction-related processes [1] points to the possibility of applying such an approach in the tasks aimed at increasing the reliability of elements and units of machines, thus leading to reduction of the uncertainty of operational decisions. At the same time the existing knowledge in the field of diagnostics enables not only the formulation of the diagnosis of a technical condition and detection of the period of cumulative wear, but also enables successful tackling of the problems related to diagnosis of early stages of development of degradation and of wear-and-tear processes. It should be stressed that numerous elements of power transmission systems of vehicles, machines and airships are subject to degradation as a result of erosion, friction, internal damping or development of cracks. One can list many such elements, including components of motors, toothed gears, valve systems and the like. The variety of phenomena, ways of diagnostic information coding as well as the big number of information carriers have contributed to the development of diversified diagnostic procedures.

An element which still remains unsolved in a satisfactory degree, even in spite of numerous achievements, is the issue of forecasting the period of time until the occurrence of a catastrophic defect. The simplest division of forecasting methods, based on vibroacoustic signals, accounts for two groups – the symptom-based and the model-supported methods. In the case of the symptom-based methods we assume that it is the results of measurements at input and output points of a system that serve as the main source of information that provides deeper insight into the process of system degradation.

Assuming that the static data characteristics do not change until events occur in the system that disturb its serviceability, the problem boils down to detection of a diagnostically-essential change caused by a defect and development of a relationship defining the moment in time when the threshold value is reached. Symptom-based methods rely on statistical and learning techniques ranging from image recognition theory, multi-dimensional statistics methods (e.g. static and dynamic analysis of principal components of PCA), linear and non-linear discrimination analysis to black-box analysis methods that rely on neural networks that self-organize according to features, as well as systems with fuzzy structures.

The main benefit of symptom-based methods is their ability to transform the multidimensional data with noise into reduced-dimension diagnostic information which is useful in the process of diagnostic-and-prognostic inference [2]. An essential drawback of this method is the heavy dependence of solution-efficiency on the quality of the system responsible for acquisition of input-output data. The most important task is to get some information, useful in the nonlinearity type discrimination.

An approach that comes to mind is the detection of the non-linearity occurring in a system by measuring and examining the non-Gaussian response of a system to a normal, random input. Then additional analyses should be conducted as regards the density of distribution of the probability of the system's response so as to determine the potential relations between the nonlinear signal transformation occurring in the system and the change of the parameters of the function describing the density of probability distribution. Multi-dimensional spectra were used for this purpose in vibroacoustic diagnosis of machines by among others Gienkin and Sokolova [3,4]. In a similar way as [5,6], they pointed to the possibility of applying the bispectrum in the analysis of statistical relations between individual components of the spectrum. The basic difficulty, which substantially restricted the application scope, was the need to find effective calculation procedures. Substantial progress in this area, in recent years, should be associated with the possibility of using multi-dimensional Fourier transforms and application of Volterra series, which facilitated the physical interpretation of the phenomena.

While analyzing the utility of cumulants for higher order spectral analysis, Mendel [7] pointed to the fact that for the multi-dimensional white noise the cumulants are multidimensional δ – functions in a similar way as the correlation of the white, uni-dimensional noise expressed by a δ – function, and that the cumulant of the sum of statistically independent random processes is equal to the sum of cumulants of individual processes. Additionally attention is drawn to the fact that up to the third order (inclusively) the moments and the cumulants are identical. The difference appears only in the case of the fourth order cumulant, which depends both on the fourth order moment and on the relevant sequence of the correlation function. Nonetheless the authors of the following publications [7,8,9] definitely give preference to the cumulant function in bispectral analysis while stressing the significance of the cumulants in the statistical evaluation of relationships.

2. USING THE BISPECTRAL ANALYSIS IN NONLINEARITY DETECTION AND IDENTYFICATION

Let us thus note that the bispectral analysis, even though it is a relatively new technique of signal analysis, enables examination of nonlinear relations between individual frequencies as well as the analysis of the impact of various instances of system non-linearity on the frequency structure of its response. In contrast with the information contained in the power spectrum, which in reality concerns the distribution of power among individual frequencies that are additionally treated as independent values, the bispectral analysis is the result of sensitivity of the third order cumulant to many other factors, including in particular the relationships between several frequencies[10]. Therefore there are several motivations behind the use of higher order spectra, but first of all the detecting and characterizing the nonlinear properties in signals as well as identifying the nonlinear systems. One of the reasons of occurrence of non-linear effects is the Quadratic Phase Coupling (QPC). QPC can thus be considered as an indicator of interaction between two harmonic components of a nonlinear process. This property can detect certain phase relations between the sum or the difference of two frequencies [9]. It is the difference between the bispectral analysis and conventional power spectrum approach, which is unable to detect the existence of QPC.

Based on the relationships presented in [11] and applied to analysis of vibroacoustic signal in [12], which rely on the bispectrum, there are many recently developed numerous procedures of detection and identification of phase coupling phenomenon. Investigating this, it is possible to write, the bispectrum in form:

$$B(f_x, f_y) = E\left[S(f_x)S(f_y)S^*(f_x + f_y)\right] \quad (1)$$

It is easy o see the bispectrum is complex and that the bispectral values depend on two frequencies f_x and f_y. Writing the equation (1) in terms of amplitude and phase quantities one becomes:

$$B(f_x, f_y) = |S(f_x)||S(f_y)||S(f_x + f_y)|e^{j\Theta_\beta(f_x, f_y)} \quad (2)$$

where $\Theta_\beta(f_x, f_y) = \Theta(f_x) + \Theta(f_y) - \Theta(f_x + f_y)$ and is called the biphase.

Using the fast Fourier transform (FFT) algorithm it is possible to calculate the raw bispectrum:

$$B_i(f_x, f_y) = S_i(f_x)S_i(f_y)S_i^*(f_x + f_y) \quad (3)$$

In addition to the basic bispectrum, the bispectrum diagonal slice is defined as:

$$B(f, f) = E\left[S(f)S(f)S^*(2f)\right] \quad (4)$$

with $f_x = f_y = f$.

Further, the summed bispectrum is defined as

$$B_\eta(f_t) = \frac{1}{\eta}\sum_{i=1}^{\eta}\left(f_{x_i}, f_{y_j}\right) = \frac{1}{\eta}\sum_{i=1}^{\eta} B\left(f_{x_j}, f_{y_i}\right) \tag{5}$$

where j = 1; 2; 3; . . . ; η.

Each Bη (f_t) represents the fraction of the total power at frequency $f_t = (f_x + f_y)$ which is due to interactive coupling at frequencies f_x and f_y. η is the number of points used on either of the f_x or the f_y frequency axes.

Generally it should be noted that the bispectral analysis focuses on the qualitative and quantitative evaluation of nonlinearity causing the effect of lack of symmetry in the function defining the probability density of the value of the analyzed diagnostic parameter or of the dynamic response in general. Many researchers [13] point to the fact that the examination of the reasons of occurrence of the effects of symmetrical nonlinearity is equally essential and they apply the analysis of the fourth order cumulants for this purpose, just as in the frequency domain of trispectrum.

The essence of tri-spectral analysis is an attempt of detecting the cubic phase coupling effect which the main factor causing the symmetrical nonlinearity. While disregarding the calculation difficulties, the element that is more important from the point of view of the diagnosis is the examination of the effect of nonlinear asymmetry which is responsible for the big values of the parameter that are hidden in the "tail" of the probability density function. It is particularly important to determine the reasons of such disturbance. Bispectral analysis may prove particularly helpful in solving this problem. Let us note that the effect of square phase coupling can be caused by a disturbance of any power, not only by the second order. Having the possibility of determining the order of the disturbance, the task of identification of the reasons, particularly the type of the developing defect, will be much easier.

3. NUMERICAL EXPERIMENT

Looking closer at the use of bi-spectral analysis for detection of various types of coupling, let us consider the example of a signal which is composed of two components: amplitude-modulated part and harmonic part of signal and which additionally contains their sums of the second and third order:

$$\begin{aligned} y(t) = A_1\left(1 + M\cos(2\pi f_r t)\right)\cdot \\ \cdot\cos 2\pi f_1 t + A_2\cos 2\pi f_2 \end{aligned} \tag{6}$$

$$x(t) = y(t) + \varepsilon\cdot y^2(t) + \sigma\cdot y^3(t) \tag{7}$$

While referring to the definition of two-dimensional analytical signal which was developed by Hahn [14], it is possibly to use only one quadratic of plane, one defined for positive value of frequency.

As could have been expected, in the case of square coupling there exist components with double base frequency as well as simple sums and differences of these frequencies. In the case of cubic part of signal, there emerge double basic frequencies as well as their sums and differences with one or the other of their components accordingly doubled. Generally the structure of bispectrum subjected to cubic coupling is more complex than the structure of the bispectrum generated by square coupling.

The results of comparative analyses that we have obtained are presented in subsequent figures.

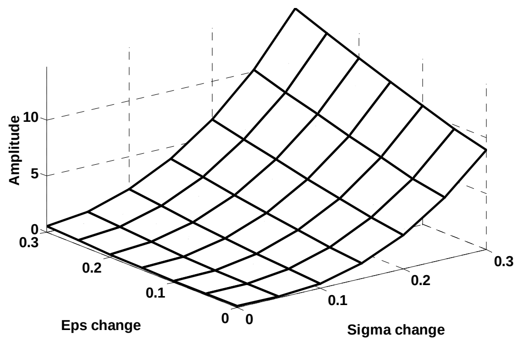

Fig.1. *$2{*}f_1{+}f_2$ – bispectrum (phase without change), $M_1{=}0$, $m_1{=}0{,}2$*

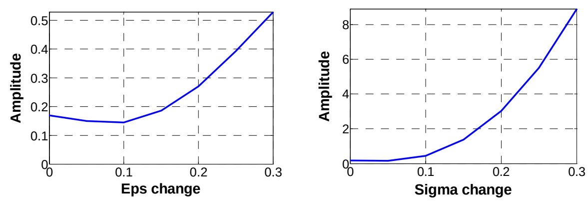

Fig.2. *Slides of two dimensional frequency plane described by formula: $2{*}f_1{+}f_2$ in function of ε and σ factor of nonlinearity, according (7)*

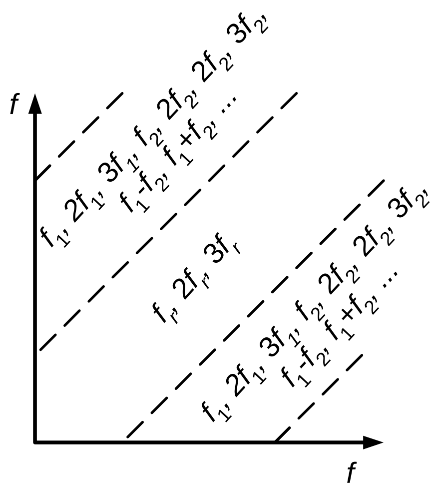

Fig.3. *Areas of two dimensional frequency plane, sensitive to special effects of different type of nonlinearity.*

The measure was built on the basis of diagonal slice bispectrum calculated also for the first quadrant. Let us note that in each case we have obtained a measure and a standard that can be used in the task of identification of the type and phase of defect development in the monitored kinematic node.

4. CONCLUSION

This paper presents the development of failures modes that explain the physical process in which a failure occured. The goal of this research is to develop methods to aid designers in making decision about the solution to avoid similar failure mode from occurring. This approach is proposed to assist with selection of monitoring techniques that will provide the greatest sensitivity to detection and rate of change of a given sympton.

Bispectral analysis enables detection of the changes of power distribution structure in a bispectral plane that are characteristic of a specify type of nonlinearity and type of defect. When the nonlinearity type can change as the defect develops, it is the possibility of identification of the measures that are particularly sensitive to such changes that becomes especially important.

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References:

[1] M. Zawisza, *Wykorzystanie informacji zawartej w sygnale wibroakustycznym do oceny prawdopodobieństwa wystąpienia awarii w przekładni zębatej*, rozprawa doktorska, Politechnika Warszawska. (*Use of Information Found in Vibroacoustic Signal for Evaluation of Probability of Occurrence of a Toothed Gear Failure*. A Ph. D. thesis, Warsaw University of Technology - in Polish) (2004).

[2] M. Jasinski, *Empiryczne modele w szczegółowej diagnostyce przekładni zębatej*, Rozprawa doktorska, Politechnika Warszawska (Empirical Models in Detailed Diagnosis of a Toothed Gear. A Ph. D. Thesis, Warsaw University of Technology - in Polish) (2004).

[3] Cz. Cempel: *Fundamentals of Vibroacoustic Dynamics of Machinery*, WNT, Warszawa (in Polish), 1982.

[4] M.D Gienkin, A.G Sokołowa.: *Wibroakusticzeskaja diagnostika maszin i mechanizmów*, Maszinostrojenije, Moskwa, 1987.

[5] Y.S Cho,. at al.: *A digital technique to estimate second-order distortion using higher coherence spectra*, Trans. on Signal Processing 40, IEEE, 1992, pp. 1029÷1040.

[6] T. Sato, K. Saslci, Y. Nakamara, *Read-time Bispectral Analysis of Gear Noise and its Application to Contactless Diagnosis*. JASA, 62(2), 1977, pp. 382÷387.

[7] J.M. Mendel: *Tutorial on Higher-Order Statistics (Spectra) in Signal Processing and System Theory: Theoretical Results and Some Applications,* Proc. of the IEEE, Vol.79 No 3, 1991, pp.278÷305.

[8] Ch. Nikias, L. Raghuveer: *Bispectrum Estimation: A Digital Signal Processing Framework,* Proc. of the IEEE, Vol. 75, No 7. July, 1987, pp.869÷891.

[9] A. Swami, J.M. Mendel, C.L. Nikias: *Higher - Order Spectral Analysis Toolbox For Use with MatLab*, The MathWorks Inc. 1995

[10] S. Radkowski, *Vibroacoustic Diagnostics of Low-Energy Failures,* ITE Warszwa-Radom. (in Polish), 2002

[11] D.-M. Yang, A. F. Stronach and P. MacConnell: *Third-Order Spectral Techniques For The Diagnosis Of Motor Bearing Condition Using Artificial Neural Networks*, Mechanical Systems and Signal Processing 2002 16(2–3), 391–4112002

[12] S.Radkowski, Z.Smalko, A.Pietak, M.Woropay, *Use of Bispectral Analysis In Condition Monitoring of Machinery*, Proceedings of the Third European Workshop, Structural Health Monitoring 2006, pp.627-634

[13] A. Rivola, *Applications of higher Order Spectra To The Machine Condition Monitoring*, Pubbl. DIEM,No. 107, Bologna 2000

[14] S.L. Hahn, *Hilbert Transforms in Signal Processing,* Artech House, Boston, London 1996