

SIMULATION MODELS OF MACHINE ELEMENTS AS COMPONENTS OF MECHATRONIC SYSTEMS

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1 Introduction

Successful products and systems are increasingly “mechatronic products and systems”, which can be seen as “smart” or “intelligent” combinations of components coming from different domains: mechanics, hydraulics/pneumatics, electrics/electronics and information processing. This certainly has an influence on the product/system development process, the methods and tools used in the process, and their basic models (see e.g. [VDI-2206]).

One aspect that is increasingly important in mechatronics is the simulation of the system behaviour in non-steady states (“dynamic simulation”): Simulation has to be based on models that span different domains, has to be applicable already in early stages of product and system development and, ideally, has to accompany the entire development process with models / model complexity “growing” as part of the process.

However, it is the mechanical components (“machine elements”) that are difficult to integrate, because their dynamic behaviour is by tradition only considered in the late stages of the development process, and usually by means of physical/layout models opposed abstract/formal/behaviour-oriented models that are used in other domains.

This paper shows how models and procedures known from electrical and control engineering also apply to mechanical components in order to get the following advantages:

- unified modelling that connects different domains
- applicable in early phases of product and system development
- with regard to their complexity, models can easily be expanded throughout the entire process of development
- in some aspects the dynamic system behaviour of (mechanical) components themselves can be explained a lot easier than by using conventional analysis methods

Our starting point is the so-called multi-port theory. Based on this theory, corresponding basic principles and rules necessary to create simulation models will be described and explained by means of a guiding example (toothed gear pair).

Parts of the basic concepts presented in this contribution were already published by the author many years ago [Weber-83, Weber-86, Weber-91]. At that time, when the term “mechatronics” was still unknown, applications were limited. Apart from that, tools for the computer-aided evaluation of (differential) equation systems were practically non-existent. In 2004 the author updated the results and extended them, putting them in the context of the dynamic behaviour of mechatronic systems. In another publication earlier this year, [Weber-05] the topic was covered with some more examples, partly from non-mechanical domains. Additionally, the considerations presented here are part of the author’s teaching activities in machine elements since 2004, which in general follow a very similar scheme as the one described and proposed by Eder (e.g. [Eder-05a]).

Before going into any details, it should be stressed that model-building with regard to the analysis/simulation of the dynamic behaviour is only one particular aspect of a much more general new view on mechanical components (machine elements): As is pointed out by Eder in [Eder-04, Eder-05a] and further developed in a contribution to the same AEDS 2005 conference [Eder-05b], machine elements, while having been explained in engineering education and having been successfully used in engineering practice for many decades, still show a “general lack of a systematic classification”. Like Eder, also the author of this contribution is convinced that a systematic approach along the lines of design science or design theory and methodology (e.g. [Hubka/Eder-92, Hubka/Eder-96, Pahl/Beitz-83, Pahl/Beitz-96, VDI-2221]) would have ample benefits both in education and in practice, and even more so when the perspective is opened from a primarily mechanical view in the past to a “mechatronic view” in the future.

In principle, the author’s ideas are very similar to Eder’s concepts, but can not be presented in detail here.

2 Machine elements as transmitters of mechanical power

Machine elements can be seen as known and proven, often standardised solution elements (assemblies, components, parts of components) that realise frequently required functions within mechanical products [Weber/Vajna-97]. Their functioning is based on certain physical principles (mainly from the mechanical domain, but also from other domains such as hydraulics and thermodynamics), sometimes in complex combinations.

In this context it is important that machine elements can also be understood as solution elements that transmit mechanical power, i.e. that transmit forces and motions. In this general sense machine elements are the constitutive components of more complex systems transmitting mechanical power, having one or more power inputs (connected to “primary movers”, “sources of mechanical power”) and one or more outputs (connected to “loads”, “consumers/drawers of mechanical power”). The term “mechanical power”, however, has to be further decomposed into its two basic values: force and velocity in the translational case and torque and angular velocity in the rotational case, respectively. Only on the level of these basic values the functions especially of machine elements can be properly described.

In the most simple case there is just one input and one output, leading to the block diagram according to **figure 1**.

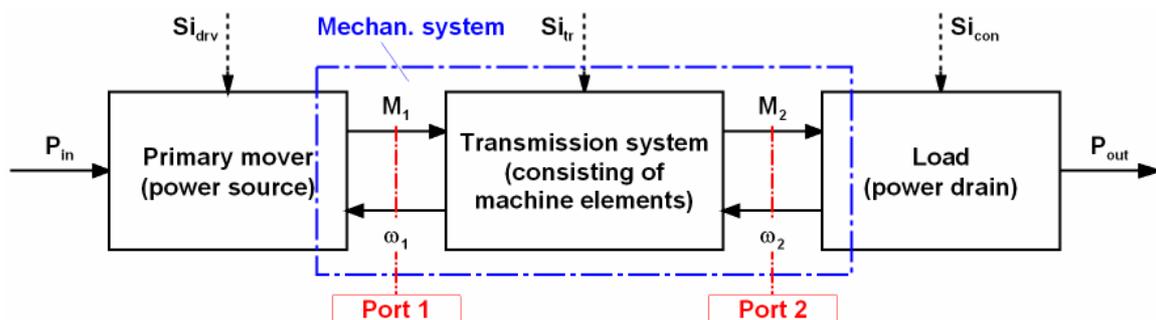


Figure 1. Machine elements as constituents of systems transmitting mechanical power (here: rotational); most simple case illustrated: two external connections (“ports”, “shafts”); one static and one kinematic degree of freedom

Using figure 1, some significant aspects can be explained:

- When you look at the block diagram, bearing in mind the context of the entire system, at every external port of a mechanical system (“shaft” or other “interface” to transmit mechanical power into or out of the system) only one of the two values – force/torque **or** motion – can be assigned, i.e. can be independent input in a logical/functional sense. The other value – again in logical/functional sense – is fed back as a result of the behaviour of the neighbouring system connected to that port, i.e. is output value.

- These findings are not at all new, they rather correspond to rules known in mechanism and transmission technology, but expressed in different terms: A mechanism has as many degrees of freedom as it has external ports (shafts or other interfaces), at every external port exactly one degree of freedom is assigned [Müller-98]. Depending on whether force/torque or motion is assigned, you can differentiate between a “static” or “kinematic” degree of freedom.
- How many of the total number of degrees of freedom of a system are static and how many kinematic depends on the structure of the transmission system itself.
- The case shown in figure 1 occurs very often: Here the transmission system (with two external ports in total) has one static and one kinematic degree of freedom, so it could be every kind of (gear) wheel transmission or belt drive without slip.
- In the field of machine elements forces and motions are not always considered at the same time: Mechanism technology often concentrates mainly on motions (purely kinematic view); in other cases (e.g. connecting elements like bolt, rivet, weld connections), motions are usually not considered. These are two special cases which are part of the general model according to figure 1: The focus is completely on kinematics if all input forces are zero (or assumed close to zero), while the purely static case occurs when all input motions are (regarded) zero.

3 Basic principles of the four-pole / multi-pole theory

3.1 Basic model and basic equations

Multi-poles are a modelling concept that is well-known in the field of electrical and control engineering, but also in acoustics (looked upon as a coupled electro-mechanical domain). They help to describe the dynamic behaviour of systems that transmit power [Oppelt-72].

The elementary model is the four-pole (sometimes called “quadrupole”), **figure 2**:

- A four-pole has two external connections (called “ports”), where power can be transported into or out of the system. In the electrical domain these are usually called “terminals”, in (rotational) mechanics we would call them “shafts”.
- Two physical values are assigned to each one of the two external ports: the so-called “effort value” or “effort variable” and the “flow value/variable”. Thus, a system having two ports has in total four external inputs and outputs or “poles”, which gives the four-pole its name.
- The multiplication of the corresponding effort and the flow variables gives the power that is transmitted at the respective port.

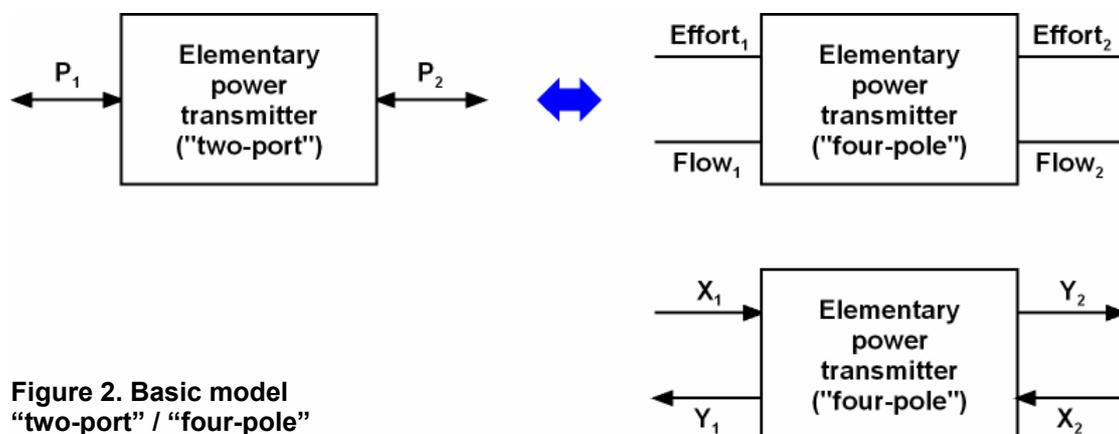


Figure 2. Basic model
“two-port” / “four-pole”

For different domains of physics (mechanical, electrical, hydraulic), different physical values being effort and flow variables can be identified, **table 1**. With regard to mechanical power it should be noted that in this contribution “force/torque” is introduced as “effort” with “velocity/angular velocity” being “flow”. It would also be possible (and would be even more correct in a physical sense) to see “force/torque = change in (angular) momentum” as the flow variable and (angular) velocity as the effort variable. The distinction between the two approaches is very interesting in theory; for further discussions on simulation models for machine elements as the focus of this paper, it is, however, of minor importance and will not be discussed any further.

Table 1. Effort and flow variables of different types of power

| | effort-, flow variable | power | work |
|------------------------------|---|---|---|
| mechanical/ translational | F, v | $P_{\text{mech, tr}} = F \cdot v = F \cdot \frac{ds}{dt}$ | $W_{\text{mech, tr}} = \int F \cdot v \cdot dt = \int F \cdot ds$ |
| mechanical/ rotational | M, ω | $P_{\text{mech, rot}} = M \cdot \omega = F \cdot \frac{d\phi}{dt}$ | $W_{\text{mech, rot}} = \int M \cdot \omega \cdot dt = \int M \cdot d\phi$ |
| electrical | $\Delta U, I$ | $P_{\text{elektr}} = \Delta U \cdot I = \Delta U \cdot \frac{dQ}{dt}$ | $W_{\text{elektr}} = \int \Delta U \cdot I \cdot dt = \int \Delta U \cdot dQ$ |
| hydraulic | $\Delta p, \dot{V}$ | $P_{\text{hydr}} = \Delta p \cdot \dot{V} = \Delta p \cdot \frac{dV}{dt}$ | $W_{\text{hydr}} = \int \Delta p \cdot \dot{V} \cdot dt = \int \Delta p \cdot dV$ |
| alternative: | $\left(\frac{\Delta p}{\rho}\right), \dot{m}$ | $P_{\text{hydr}} = \frac{\Delta p}{\rho} \cdot \dot{m} = \frac{\Delta p}{\rho} \cdot \frac{dm}{dt}$ | $W_{\text{hydr}} = \int \frac{\Delta p}{\rho} \cdot \dot{m} \cdot dt = \int \frac{\Delta p}{\rho} \cdot dm$ |
| thermal | $\Delta T, \dot{S}$ | $P_{\text{therm}} = \Delta T \cdot \dot{S} = \Delta T \cdot \frac{dS}{dt}$ | $W_{\text{therm}} = \int \Delta T \cdot \dot{S} \cdot dt = \int \Delta T \cdot dS$ |

In a logical/functional sense only one of the two physical values can be input with regard to every external port (i.e. can be determined by the neighbouring external system connected to that port, **X**), the other one is output (i.e. is impressed onto the system connected to that port, **Y**). Whether, in a particular case, at a particular port flow or effort is input depends on the structure of the transmission system as well as on how it is integrated into the whole system environment.

In the field of electrical, control and acoustic engineering it is common practice to assume only linear behaviour, i.e. linear equations that connect the system’s external and internal values. The linear four-pole defined in this way has the generic structure illustrated in **figure 3**.

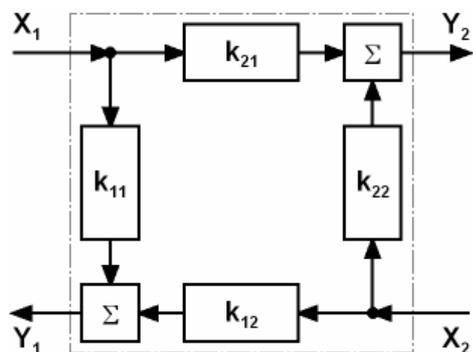


Figure 3. Generic structure of a linear four-pole

It is worth mentioning that the structure shown in figure 3 as well as all other block diagrams discussed in this paper differ from figures normally used in control engineering in one point: The summation/superposition of values is indicated by a rectangular box with the symbol “Σ” in it and not – as in the field of control engineering – by means of a small circle and a “plus” sign (“⊕”). This is done to avoid confusion with “branching-off points” where a value is distributed to several branches without changing its quantity (see discussion in section 4.2).

The system behaviour (transfer of input to output values) of the generic linear four-pole according to figure 3 can be described as:

$$Y_1 = k_{11} \cdot X_1 + k_{12} \cdot X_2 \quad (1)$$

$$Y_2 = k_{21} \cdot X_1 + k_{22} \cdot X_2 \quad (2)$$

Hence as a matrix:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \underline{\underline{K}} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (3)$$

The assumption of linearity is no strict necessity, it is just common practice in other domains in which the four-pole approach is widely used. When considering non-linear cases, in equations (1) and (2) the linear mathematical functions with their constants k_{ij} have to be replaced by general functions of the form $f_{ij}(X_j)$ – principally any function appropriate to describe the respective physical effect is possible: polynomials, trigonometric functions, However, in this case a matrix form according to equation (3) is not applicable anymore, since matrices can only capture linear relations.

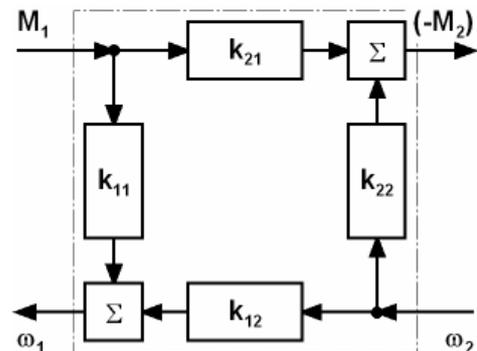
In the field of mechanical systems non-linear relations are needed quite often. One example is the description of sliding friction effects – the Coulomb-type “dry” friction as well as the Newton-type “fluid” friction effect. Here is no space to start a detailed theoretical discussion of the non-linear case; in this paper non-linear considerations will be taken up again “ad hoc”, where appropriate. However, since matrix representations as shown in equation (3) are strictly confined to the linear case, they are not used any longer in this paper.

3.2 Mechanical four-poles

Starting from figure 3, a general four-pole becomes a mechanical four-pole if the generalised input and output values (X_i, Y_j) are defined as mechanical inputs and outputs, i.e. forces and velocities (translational case) or torques and angular velocities (rotational case).

The mechanical system illustrated in figure 1 – transmitting rotational mechanical power with just two external power ports (“shafts”) and one static and one kinematic degree of freedom – turns into the mechanical four-pole shown in **figure 4**, provided that the transfer function is again (assumed) linear.

Figure 4. Structure of a linear *mechanical* four-pole (here: transmission of rotational mechanical power)



In all diagrams of mechanical systems where an effort value (force or torque) is output in a logical/functional sense (see M_2 in figure 4), a minus symbol will be added in order to reverse the sign. This ensures that all equations derived from the respective four-pole structures automatically deliver “physically correct” signs:

- By convention, in mechanical analyses the “correct” directions of all force values are defined in such a way that the effects of neighbouring systems onto the analysed system are considered (thus, the analysed system will always present itself in a state of equilibrium of forces).
- When it comes to system simulation, this could also be done completely different, i.e. by defining all force values in such a way that the effects from the logical/functional origins onto the next neighbouring systems are captured (i.e. considering forces “along the directions of the arrows”). As a result you would also get coherent descriptions of the

system behaviour; however, all force values that are logical/functional outputs would have reversed signs compared to the standard case.

Bearing in mind this rule of signs of output effort values, instead of the general equations (1) and (2), you get the following specific equations as descriptions of the system behaviour of the (linear) mechanical four-pole according to figure 4 (rotational case). The presentation in form of a matrix as in equation (3) is left out for the reasons mentioned above.

$$\omega_1 = k_{11} \cdot M_1 + k_{12} \cdot \omega_2 \quad (4)$$

$$(-M_2) = k_{21} \cdot M_1 + k_{22} \cdot \omega_2 \quad (5)$$

3.3 Four-poles without power losses (with non-dissipative behaviour)

By using four-pole models like the ones shown in figures 3 and 4, some general questions with regard to the behaviour of the described systems can be discussed on a quite formal level, i.e. without having to go into physical or even layout details.

One of the most interesting questions is: Which are conditions where the system has a behaviour free of power losses? This shall be investigated for all operating conditions, i.e. for all possible input values (X_1, X_2) or, in the mechanical case according to figure 4, (M_1, ω_2).

In the mechanical case powers P_1 and P_2 can be calculated as follows:

$$P_1 = M_1 \cdot \omega_1 = M_1 \cdot (k_{11} \cdot M_1 + k_{12} \cdot \omega_2) \quad (6)$$

$$P_2 = M_2 \cdot \omega_2 = -(k_{21} \cdot M_1 + k_{22} \cdot \omega_2) \cdot \omega_2 \quad (7)$$

No power losses / no dissipation means:

$$P_1 + P_2 = k_{11} \cdot M_1^2 + (k_{12} - k_{21}) \cdot M_1 \cdot \omega_2 - k_{22} \cdot \omega_2^2 = 0 \quad (8)$$

If we want equation (8) to be valid for all possible input values (M_1, ω_2), then the following conditions must be fulfilled:

$$k_{11} = k_{22} = 0 \quad (9)$$

$$(k_{12} - k_{21}) = 0 \quad (10)$$

Interestingly enough, the two conditions (9) and (10) immediately lead to conclusions about the "allowed" inner structure of (linear) four-poles and, subsequently, about the necessary structure of corresponding real systems: As a general rule, dissipation-free power transmitting systems have to have two separate structural branches to transfer input values into output values. **Figure 5** illustrates this finding for the mechanical four-pole according to figure 4:

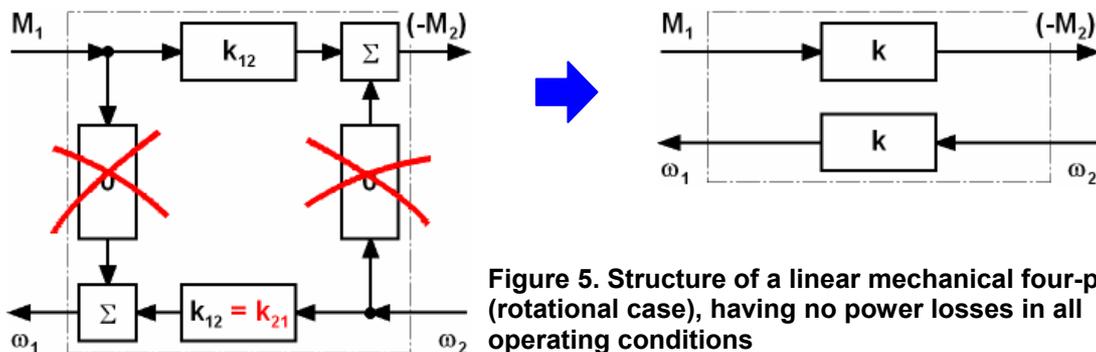


Figure 5. Structure of a linear mechanical four-pole (rotational case), having no power losses in all operating conditions

Some interesting further conclusions can be drawn from the discussion about non-dissipative behaviour of power transmissions:

- Power transmission with good efficiency ratio throughout the whole operating range is not a quality influenced primarily by physics, but rather influenced by the inner structure of the system.
- Many machine elements are, in fact, constructed in a way that follows the ideal structure illustrated in figure 5: All positive locking gear trains and clutches transmit forces/torques decoupled from motions, consequently the “static” and “kinematic” aspects can be considered separately.
- The same is true vice versa: In all cases in which power losses occur (or are even part of the required system behaviour: e.g. clutches and gears in the state of slip, brakes), the static values are **not** decoupled from the kinematic values. This will be further examined in the following chapter 4.

3.4 Catenation of four-pole models

Four-pole models can be easily combined (catenated) to build models of more complex systems, **figure 6, top**. However, you have to consider that even in the completely linear case the overall mathematical transfer function can **not** simply be calculated by multiplying the matrices (see equation (3)) of the catenated four-poles. The reason is that the two logical/functional output values of each elementary four-pole as well as the overall output values of the whole system (in general: $(\mathbf{Y}_1, \mathbf{Y}_2)$; in the case of the mechanical [rotational] four-pole: $(\boldsymbol{\omega}_1, \mathbf{M}_2)$) belong to **different** external ports (external “shafts” in the case of rotational mechanical power).

Instead, the overall transfer functions are found out by means of “calculating backwards”: Start from the output values (that are connected to different ports!), follow them up across all the different inner elements of the structure, until the input values (that are also connected to different ports) are reached, see **figure 6, bottom**.

Only a “mathematical trick” (and a quite complicated one, at that) could describe the catenation by means of a multiplication of the matrices of the partial systems: 1 – transform all catenated four-pole models into substitute models in which both values at every port have the same logical/functional direction; 2 – multiply the substitute matrices; c – re-transform the result back to a state with one input and one output at each port afterwards. This procedure was already discussed in [Feldmann-71] for hydraulic and hydro-mechanical systems, but can be applied generally. It will not be described here in detail.

3.5 Multi-pole models

“Two-ports” or rather “four-poles”, which describe systems with two external power ports, can be extended to so-called “multi-ports” (sometimes called “n-poles”) by adding further external ports (more than two external “shafts”, “terminals”, ...). A gearbox that has three external shafts is, for instance, a “three-port” or a “six-pole” (three external shafts \Rightarrow six external input and output values in total, three each static and kinematic). For multi-poles similar structures, rules and relations apply as for elementary four-poles. This will not be discussed here in further detail. In [Weber-05] the multi-pole model of an epicyclic gear system is explained as an example.

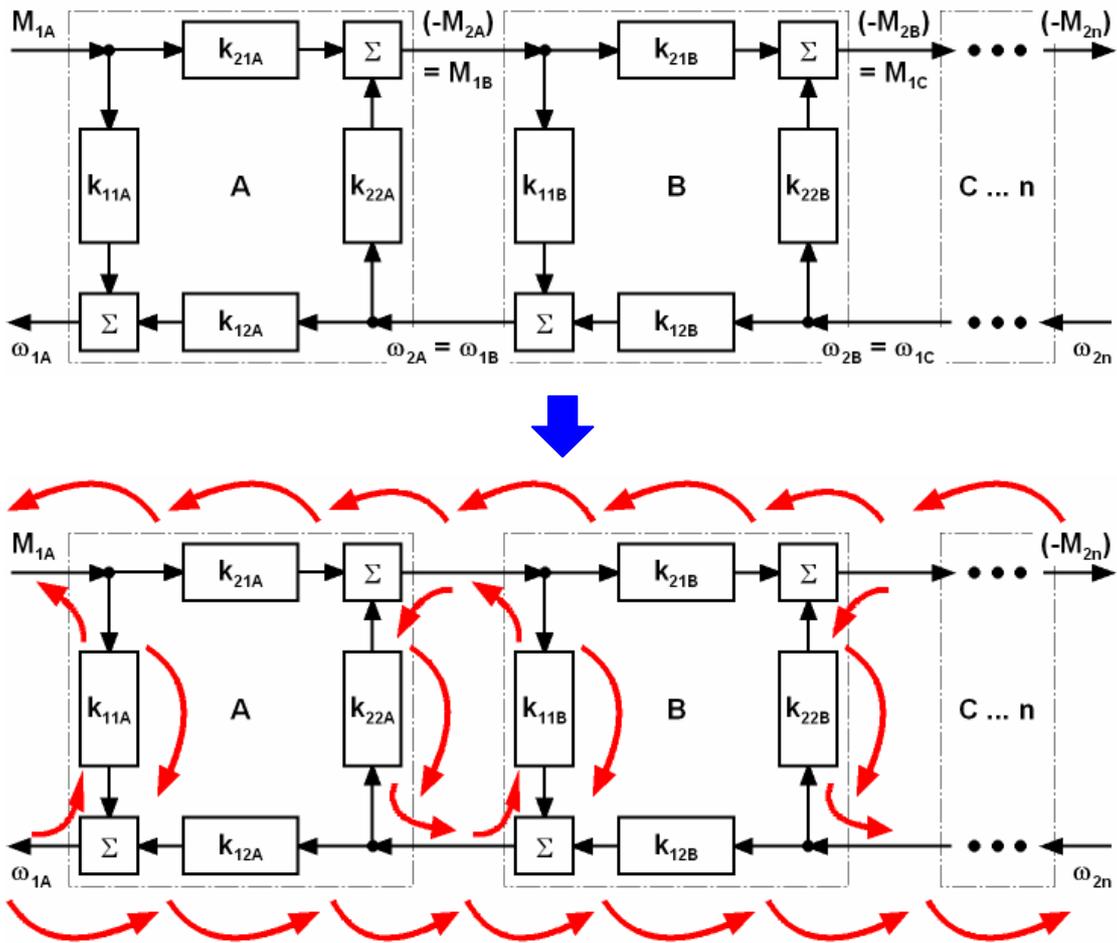


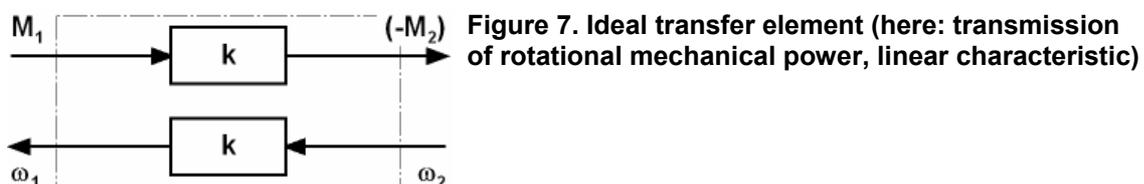
Figure 6. Catenation of four-pole models and determining the overall mathematical transfer function (scheme, shown for the example of transfer of rotational mechanical power)

4 Element types of multi-pole models

In multi-pole models (as in all other similar models, see [Wellstead-79, Breedveld-84, Karnopp et al-00]) there can only occur three basic types of elements, which will be introduced and explained in this section.

4.1 Ideal transfer elements

Ideal transfer elements have the structure of a non-dissipative four-pole, i.e. a four-pole made up of two separate branches (see figure 5, right). When we talk about a mechanical four-pole – particularly one that transmits rotational mechanical power – under the conditions that one branch connects the effort variables while the second (separate) one “transports” the flow variables and if we keep up the assumption of linear transfer characteristics you get the model shown in **figure 7**.

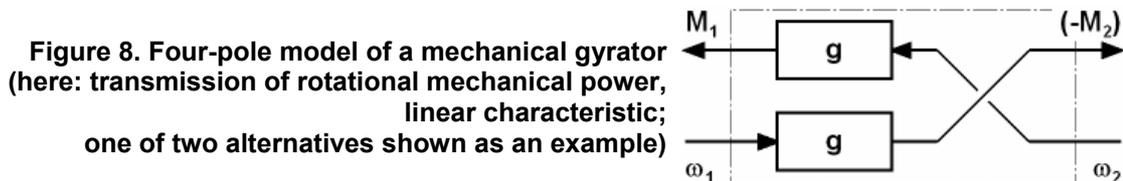


In systems theory, elements of this kind are called “ideal transformers” or shortly “TF-elements”. Examples in the mechanical domain – here with connected input and output values of the same type (as in figure 7: torque connected to torque and angular velocity to angular velocity) – are all non-dissipative translators of forces (e.g. levers) which at the same time are always motion translators, as well as all positive gear trains which also transmit forces/torques and (angular) motions simultaneously but separately.

If values out of different physical domains are connected, but still under the condition that effort is connected to effort and flow to flow, then in systems theory the term “ideal transducer” is used. Examples from the mechanical domain are all transformers of rotational to translational values (e.g. pulleys, rack and pinion gears) as long as we assume them to be non-dissipative. Beyond mechanics, hydrostatic pumps and motors are examples for ideal transducers, again if assumed free of power losses.

Since the system behaviour is non-dissipative, ideal transfer elements are also called “power-conserving two-ports”.

Apart from transformers and transducers, however, there is another category of ideal transfer elements, the so-called gyrators (“GY-elements”). They also have two completely decoupled structural branches; these branches connect, however, the effort value of one port with the flow value of the other port or vice versa. **Figure 8** displays one of two possible gyrator structures in the field of mechanics.



While gyrators play an important role in electrical engineering, they are rarely found in the mechanical domain: gyroscopes are the only example for mechanical gyrators (at the same time giving this special element type its name). For this reason this category of ideal transfer elements is not discussed here in further detail.

According to figure 7 you get the following equations when describing the system behaviour of ideal (mechanical) transfer elements in the linear case:

$$(-M_2) = k \cdot M_1 \quad (11)$$

$$\omega_1 = k \cdot \omega_2 \quad (12)$$

It is important to mention (again) that a linear transfer behaviour is only a special, albeit often used case and that also non-linear relations can occur quite frequently (e.g. polynomials, even trigonometric functions).

4.2 Junction elements

Junction elements are used to model connections of more than two ports (e.g. connecting more than two “shafts” in the case of transmitting rotational mechanical power). In the elementary case, exactly three ports are coupled without additional changes of the transmitted values. For this kind of elementary “three-ports” or “six-poles” there are exactly two alternative concepts:

- a. summation/superposition of flow variables, accompanied by identical effort variables
- b. summation/superposition of effort variables, accompanied by identical flow variables

Figure 9 shows on the left and right side the corresponding block diagrams of the elementary junction element types according to concepts a and b, respectively. Again, the case of transmitting (rotational) mechanical power is specified here as an example, other cases are

analogous. The summation/superposition of values in one branch of the structure (illustrated by a rectangular box with the symbol “ Σ ”) is connected inseparably to the coupling of three identical values in the other branch of the structure; this is displayed in the block diagrams by a “branching-off point” in form of a filled node. It is very important to avoid confusion between the “summation/superposition” part and the “branching-off” part of junction elements; therefore, the author introduces here the notation displayed in figure 9 which differs slightly from the usual notation in control engineering.

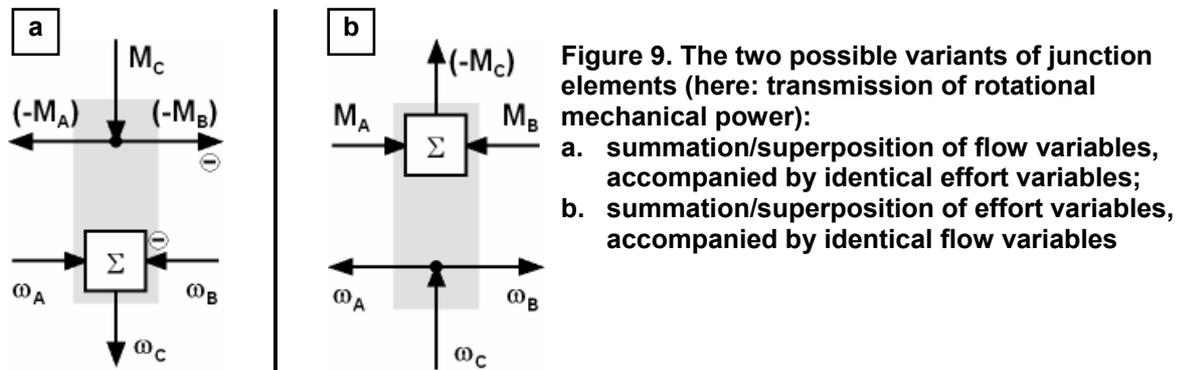


Figure 9. The two possible variants of junction elements (here: transmission of rotational mechanical power):

- a. summation/superposition of flow variables, accompanied by identical effort variables;
- b. summation/superposition of effort variables, accompanied by identical flow variables

The summation/superposition of the three values in one branch of both types of junction elements requires that two of them have to be logical/functional inputs (have to be assigned by neighbouring elements or systems); otherwise, the sum can not be evaluated. Accordingly, the coupling of three identical values in the other branch of the respective junction element requires only one of them to be input (to be defined from outside the element).

Therefore, the junction element according to concept a (figure 9a) has in the mechanical case only one static but two kinematical degrees of freedom (i.e. one force/torque but two motions have to be assigned by neighbouring elements or systems). In contrast to that the junction element according to concept b (Figure 9b) requires two static but only one kinematic degrees of freedom (i.e. two force/torque values but only one motion have to be specified by the external connected elements or systems).

The mutual dependence of the outputs on the inputs can be mathematically expressed in the following way¹:

$$\text{a. } (-M_A) = M_C = M_B \quad (13)$$

and

$$\omega_C = \omega_A - \omega_B \quad (14)$$

$$\text{b. } (-M_C) = M_A + M_B \quad (15)$$

and

$$\omega_A = \omega_B = \omega_C \quad (16)$$

Examples for junction elements according to concept a (Figure 9a) from the mechanical domain are the relative movement of two bodies with (identical) contact force, in electronics and hydraulics it is the parallel connection of three electrical/hydraulic units.

¹ Note on signs: In the block diagrams in figure 9 as well as in equations (13) to (16) the signs are chosen according to the sign rules for forces as logical/functional output values of mechanical systems which were explained in chapter 3. In the case of the junction element according to concept a (figure 9a) this requires an additional reversion of one sign, marked by “ \ominus ”.

A mechanical example for junction elements according to concept b (Figure 9b) is the sum of all forces/torques acting on a moving (rigid) body. In electronics and hydraulics it is the series connection of three electrical/hydraulic units.

Both junction element types in Figure 9 are “power-conserving”. This is not a surprising fact as they have – like the ideal transfer elements treated in section 4.1 – a structure with two completely decoupled branches.

In systems theory, the junction element according to concept a (Figure 9a) is also called a “0-junction”, the junction element according to concept b (Figure 9b) is a “1-junction”. These denominations result from another modelling method that is not described in this paper, namely that of Bond graphs [Wellstead-79, Karnopp et al-00]. This topic will not be explored in further detail here.

Finally, please note that the junction elements explained here have already been implicitly used in the elementary four-pole models in figures 3 and 4 in order to add up the influences of both inputs on the outputs.

4.3 “Cross-coupling elements”

In this contribution the term “cross-coupling elements” denotes structural elements in four-pole/multi-pole models which directly connect interrelated effort and flow values (in the mechanical case: force/torque directly coupled to motion). As already discussed in section 3.3 these elements have a special significance as they cause (at least temporary) power losses. There are three different types of “cross-coupling elements” which differ from each other as they connect effort values with flow values of different temporal derivations. In the mechanical case these are the following possible “cross-couplings”:

- a. Coupling acceleration/angular acceleration (the derivation of velocity/angular velocity with respect to time) to force/torque: inertia-induced force/torque (“masses and fly-wheels”)
- b. Coupling velocity/angular velocity directly to force/torque: resistance-induced force/torque (“friction and damping elements”)
- c. Coupling displacement/angular displacement (the integration of velocity/angular velocity with respect to time) to force/torque: elasticity-induced force/torque (“springs”)

These three types of “cross-coupling elements” will be explained in the next sub-sections in terms of four-pole structures and equations.

Elements describing inertia effects

Figure 10 shows sketches as well as related four-pole diagrams for (mechanical) “cross-coupling elements” of the type a (coupling acceleration/angular acceleration to force/ torque), with the translational case in the left and the rotational case in the right column of the figure².

In principle, in the translational case an acceleration (= derivation of velocity with respect to time) via the inertia causes a force difference (ΔF_p); accordingly, in the rotational case an angular acceleration (= derivation of angular velocity with respect to time) via the moment of inertia causes a torque difference (ΔM_p). Depending on the sign of the (angular) acceleration this difference is either positive or negative. If the (angular) acceleration equals zero the inertial force/torque also is zero.

If the (angular) acceleration does not equal zero and, consequently, the force/torque difference is also non-zero, then the powers carried at the external ports of the element (translational case: $P_1 = F_1 \cdot v_1$ and $P_2 = F_2 \cdot v_2$; rotational case: $P_1 = M_1 \cdot \omega_1$ and $P_2 = M_2 \cdot \omega_2$) don't sum up to zero anymore: The element has – depending on either a minus or plus sign of the difference of power – temporary power losses or power gains.

In terms of energy balances, this can be explained by a certain amount of the input power being (temporarily) used to increase or decrease the kinetic energy stored in the element.

² In the block diagrams in Figure 10 as well as in all following diagrams the signs are chosen according to the sign rules for force values as outputs of mechanical systems, as explained in chapter 3.

Thermodynamically, this is a reversible process (see corresponding arrows in the block diagrams: because of reversibility on the conversion both directions possible).

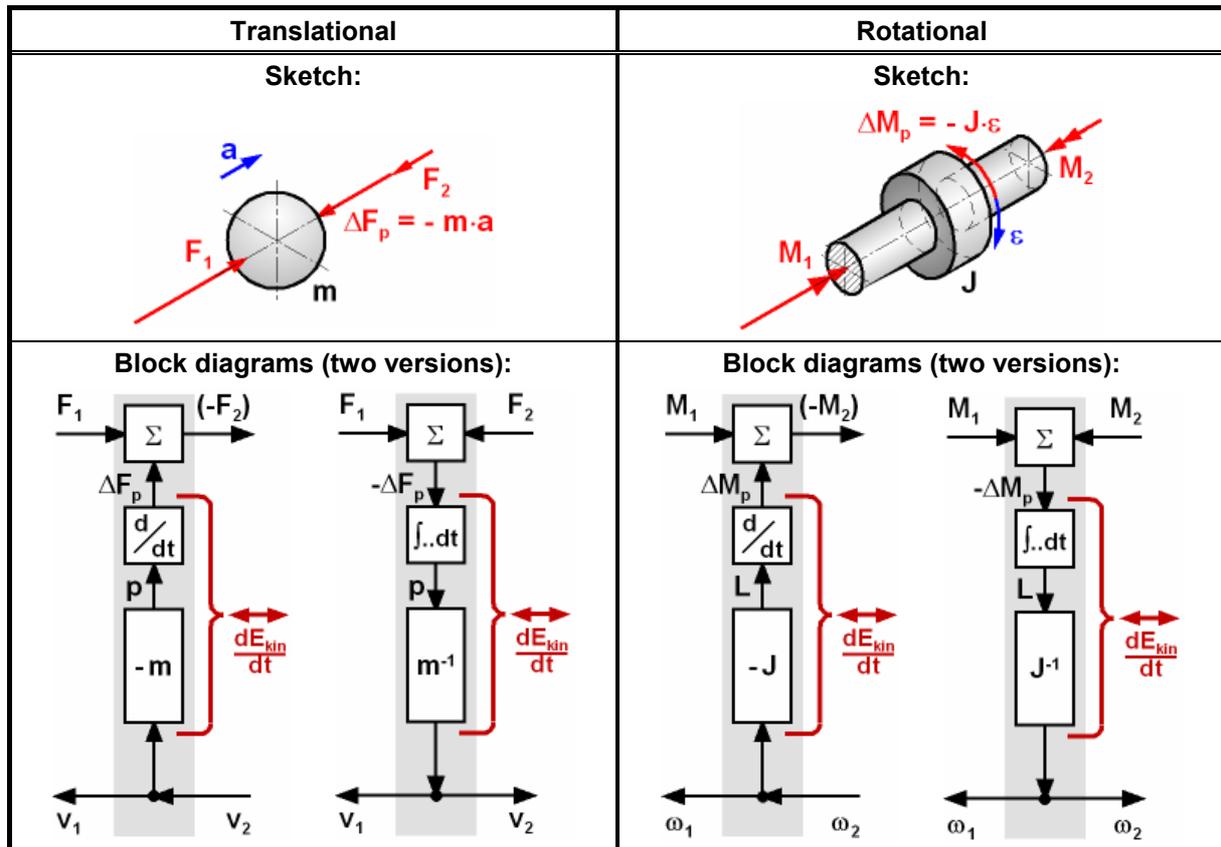


Figure 10. “Cross-coupling element” acceleration / angular acceleration ↔ force / torque

Both for the translational and the rotational case, there are two versions of four-pole block diagrams displayed in figure 10; they describe two different operating modes that result from the integration of the “cross-coupling element” discussed here into the neighbouring system environment:

- In the first version (displayed left in the two columns of figure 10) the (angular) velocity state (in general terms: the flow variable) is impressed on the element by one of the external neighbouring elements or systems. If the motion state changes in time, then the inertia induces a (positive or negative) force/torque difference between input and output on the side of the effort variables.
- In the second version (displayed right in the two columns of figure 10) at both external ports of the element the forces or torques, respectively, are determined by the neighbours. Then, if there is a non-zero difference between the two external forces/torques (ΔF_p or ΔM_p , respectively), due to the effects of inertia this will be converted into an (angular) acceleration or deceleration, i.e. a change of the velocity state of the element which, in turn, is impressed on the connected neighbouring systems. In this version the internal logical/functional flow inside the “cross-coupling element” is reversed compared to the first version. Therefore, what was a derivation element (d/dt) has now to become an integration element ($\int \dots dt$).

The behaviour (transfer of input to output values) of the “cross-coupling element” shown in figure 10 can be described by the following equations (only version one of the two operating options noted):

Translational:

$$(-F_2) = F_1 + \Delta F_p = F_1 + \frac{dp}{dt} = F_1 + \frac{d}{dt}(-m \cdot v_2) \quad (17)$$

Rotational:

$$(-M_2) = M_1 + \Delta M_p = M_1 + \frac{dL}{dt} = M_1 + \frac{d}{dt}(-J \cdot \omega_2) \quad (18)$$

The equations for the real-world “cross-coupling elements” are in the field of traditional (non-relativistic) physics always linear.

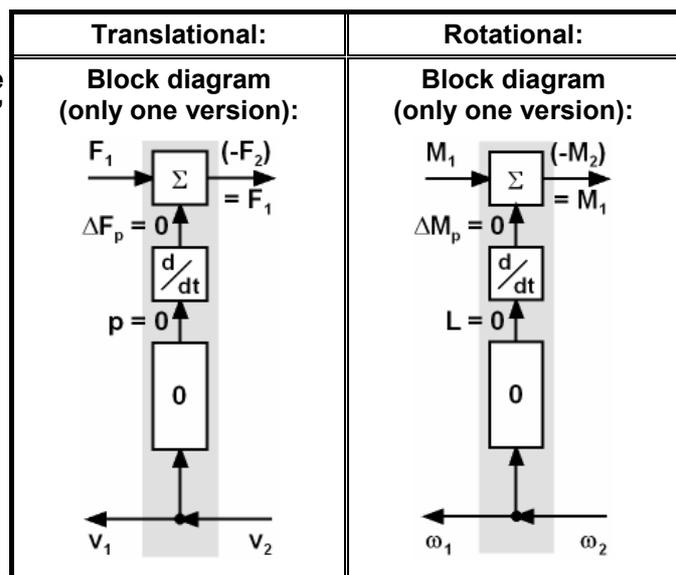
However, in special cases the inertia or moment of inertia can vary during operation (e.g. in vehicles that loose weight due to the consumption of carried fuel). Therefore, the correct placement of the derivation element (d/dt) in the structures is **behind** the element which represents inertia or moment of inertia, respectively. Or to express it in a physical way: The force/torque difference (ΔF_p or ΔM_p , respectively) is in fact the result of change in the (angular) momentum rather than the result of a change of the (angular) velocity.

In systems theory, “cross-coupling elements” of the type displayed in figure 10 are called “I-Elements” (“inertia” or “induction” elements). They are in the general case (in non-steady-state, “dynamic” conditions) not “power-conserving”, but still “energy-conserving”.

In terms of the structure, please note that the inner part of the “cross-coupling element” displayed in figure 10 and also of all other types of “cross-coupling” elements that will be explained in the following sub-sections are in systems theory occasionally known as “one-ports” or “two-poles”. However, they can not exist alone; in fact, they have to be connected with accessible external ports by means of junction elements (in this case according to type b explained in section 4.2) so that the complete structure is again a “two-port” or “four-pole”.

For “cross-coupling elements” according to figure 10 there is a special case which has some importance especially in mechanics: This is a “mass-less” or “inertia-less” element having an inertia equal (or assumed equal) to zero. This special case is formally described by the condition $m = 0$ or $J = 0$, respectively, and is displayed in **figure 11** in form of a four-pole structure.

Figure 11. Special case “element with zero inertia”



Elements describing resistance effects

Figure 12 shows sketches and related four-pole diagrams for (mechanical) “cross-coupling elements” of the type b (coupling directly velocity/angular velocity to force/torque). Again, the translational and the rotational case are displayed separately (left or right column of figure 12, respectively).

In principle, a difference in relative (angular) velocity between the two ports of the element causes a resistance force or torque which, in turn, is equally evident at both ports.

The powers at both external ports (translational case: $P_1 = F_1 \cdot v_1$ and $P_2 = F_2 \cdot v_2$; rotational case: $P_1 = M_1 \cdot \omega_1$ and $P_2 = M_2 \cdot \omega_2$) do not sum up to zero (as long as the velocity-difference is non-zero), the element always has power losses.

In terms of energy balances, this can be explained by the fact that a certain amount of the input power is dissipated into thermal power which is an irreversible process (see the corresponding arrows in the block diagrams: because of the conversion being irreversible only one direction possible).

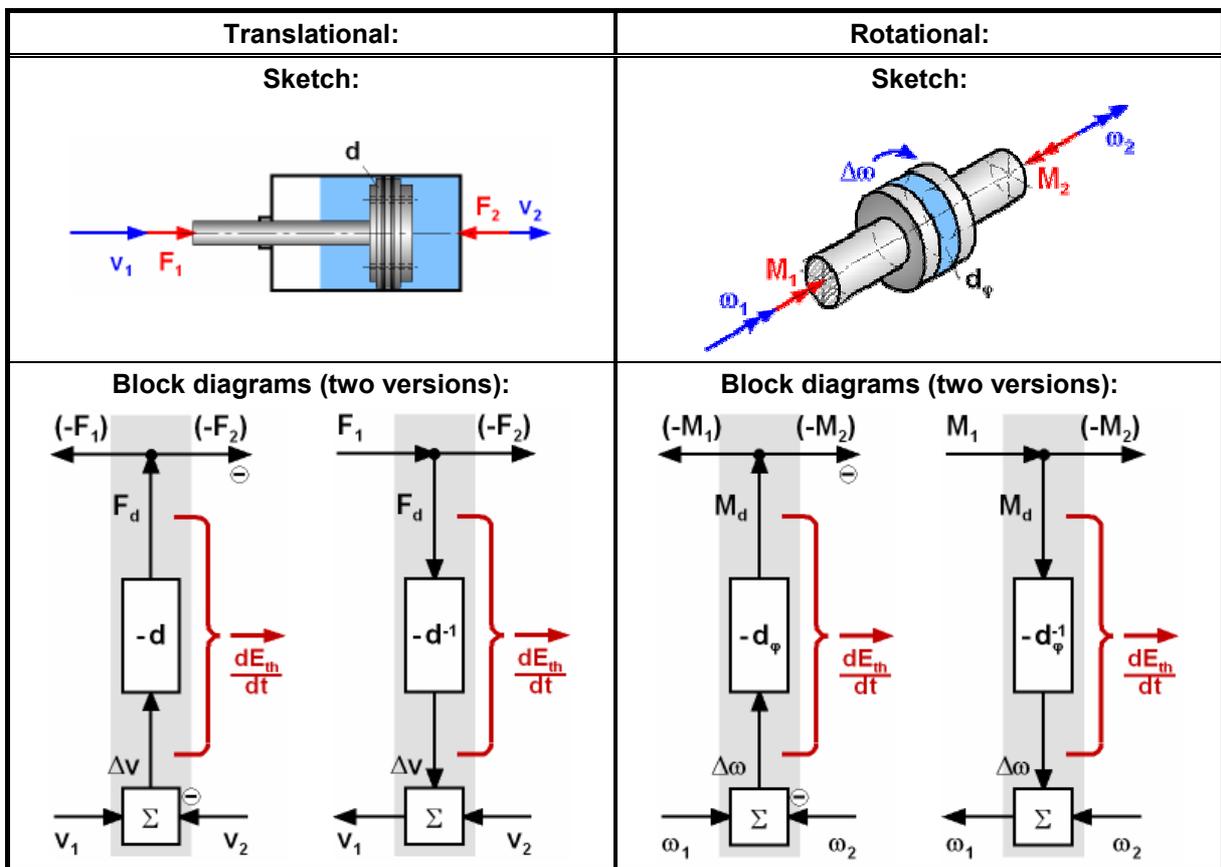


Figure 12. “Cross-coupling element” velocity / angular velocity ↔ force / torque

Again, for the translational as well as the rotational case there are two versions of four-pole block diagrams displayed in figure 12 describing two different operating modes that depend on the integration of the element into its environment:

- In the first version (displayed left in the two columns of figure 12) at both ports the velocities are assigned by the external neighbouring systems. The difference between these velocities causes the resistance force or torque, respectively, (F_d , M_d) which at the same time affects both external ports.
- In the second version (displayed right in the two columns of figure 12) an external force is impressed on the element. The only way the element can react on this is by adjusting the relative (angular) velocity (Δv , $\Delta \omega$), i.e. by increasing or decreasing the slip accordingly.

Again, the two versions display a reversed internal logical/functional flow inside the “cross-coupling element”.

For the structures shown in figure 12 the following equations can be noted (in each case for the first of the two operating versions) which, for reasons of simplicity, assume a linear behaviour of the “cross-coupling”:

Translational:

$$-(-F_2) = (-F_1) = F_d = (-d) \cdot \Delta v = (-d) \cdot (v_1 - v_2) \quad (19)$$

Rotational:

$$-(-M_2) = (-M_1) = M_d = (-d_\varphi) \cdot \Delta \omega = (-d_\varphi) \cdot (\omega_1 - \omega_2) \quad (20)$$

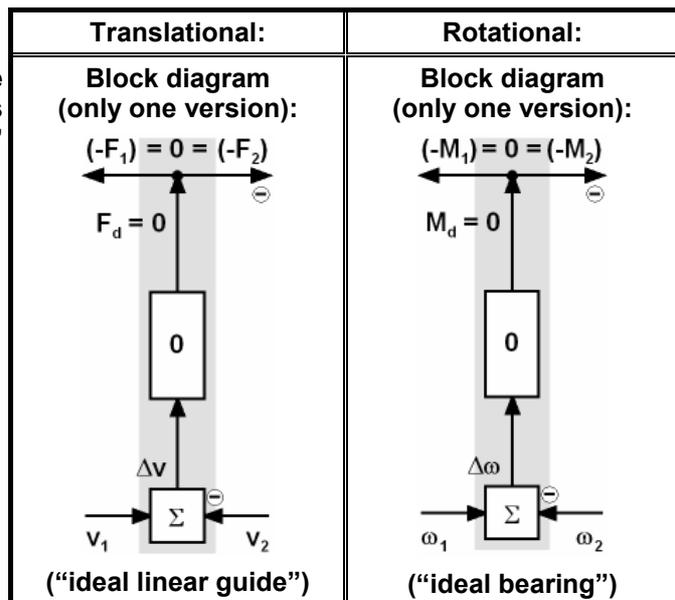
Especially in the case of mechanical resistances a linear behaviour hardly occurs in reality. Even the most simple Coulomb (“dry”) friction, very widely used as approach to describe mechanical resistances, has to be described through another function (signum function). The so-called Newtonian (“fluid”) sliding friction has an even more complex behaviour that has to be described depending on precise geometry, flow and operating conditions (including, for example, the temperature!). Therefore, the linear dependencies given in equations (19) and (20) can only be seen as a theoretical special case and have to be replaced by appropriate non-linear mathematical relations in order to properly describe "realistic" cases.

In contrast to mechanics, in the electrical domain we can usually describe electric resistances by a linear behaviour with good approximation (Ohm’s law).

In systems theory, “cross coupling elements” of the type displayed in figure 12 are called "R-elements" (resistance elements). They are neither “power-conserving” nor “energy-conserving” but rather "power and energy-dissipating two-ports".

For “cross coupling elements” according to figure 12 there is also a special case which sometimes is considered in mechanics: This is the "relative movement without friction" or, in terms of real components, the “ideal linear guide” or “ideal rotational bearing”. This special case is formally defined by the condition $d^{-1} = 0$ or $d_\varphi^{-1} = 0$, respectively; the corresponding four-pole structure is shown in **figure 13**.

Figure 13. Special case
“element allowing relative movements
without resistance/friction”



Elements describing elasticity effects

Figure 14 shows the sketches and related four-pole diagrams for (mechanical) “cross-coupling elements” of the type c (coupling displacement/angular displacement to force/ torque). As in the cases described in the previous two sub-sections, the translational and the rotational cases are displayed separately (left or right column of figure 14, respectively).

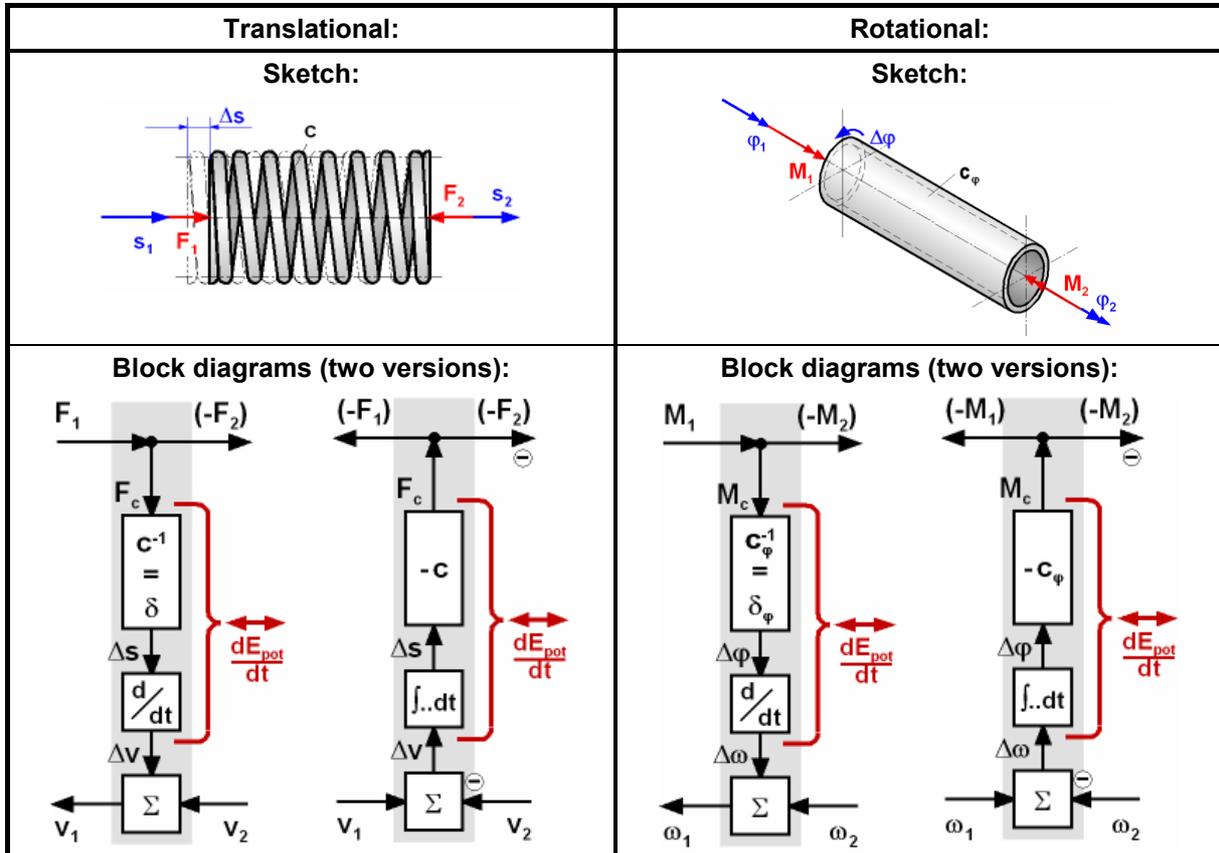


Figure 14. “Cross-coupling element” displacement / angular displacement ↔ force / torque

In principle, a force or torque impressed on an elastic body causes a deformation (= relative displacement or relative angular displacement, respectively). If this varies in time, then there is a (temporary) difference between the (angular) velocities of the two ports. The forces or torques at the two ports must, however, be identical for reasons of equilibrium.

When changes of external forces/torques occur, hence also changes in the deformation state, and accordingly temporary differences of (angular) velocities, then the powers carried at the external ports of the element (translational case: $P_1 = F_1 \cdot v_1$ and $P_2 = F_2 \cdot v_2$; rotational case: $P_1 = M_1 \cdot \omega_1$ and $P_2 = M_2 \cdot \omega_2$) don't sum up to zero: The element has – depending on either a minus or plus sign of the difference of power – temporary power losses or power gains.

In terms of energy balances, this can be explained by a certain amount of the input power being (temporarily) used to increase or decrease the static energy stored in the element. Thermodynamically, this is a reversible process (see corresponding arrows in the block diagrams: because of reversibility on the conversion both directions possible).

Again, for the translational as well as the rotational case there are two versions of four-pole block diagrams displayed in figure 14 describing two different operating modes that depend on the integration of the element into its environment:

- In the first version (displayed left in the two columns of figure 14) the external force is impressed on the element (equal at both ports for reasons of equilibrium) by the

connected external elements or systems. If the force changes in time, a change in deformation and a (temporary) difference of (angular) velocities between the ports will occur.

- In the second version (displayed right in the two columns of figure 14) the two external neighbouring systems will assign (temporarily) different velocities or angular velocities, respectively, to the two ports of the element. The only way the element can react is by increasing or decreasing its elastic deformation state (as long as it does not break!) which, in turn, results in an increase or decrease of the force/torque at the two external ports.

The behaviour of the element shown in figure 14 can be expressed mathematically by the following equations (in each case for the first of the two operating versions), here again assuming a linear elastic characteristic for reasons of simplicity:

Translational:

$$\mathbf{v}_1 = \mathbf{v}_2 + \Delta \mathbf{v} = \mathbf{v}_2 + \frac{d}{dt}(\Delta \mathbf{s}) = \mathbf{v}_2 + \frac{d}{dt}(\mathbf{c}^{-1} \cdot \mathbf{F}_c) = \mathbf{v}_2 + \frac{d}{dt}(\mathbf{c}^{-1} \cdot \mathbf{F}_1) \quad (19)$$

Rotational:

$$\omega_1 = \omega_2 + \Delta \omega = \omega_2 + \frac{d}{dt}(\Delta \varphi) = \omega_2 + \frac{d}{dt}(\mathbf{c}_\varphi^{-1} \cdot \mathbf{M}_c) = \omega_2 + \frac{d}{dt}(\mathbf{c}_\varphi^{-1} \cdot \mathbf{M}_1) \quad (20)$$

The equations (19) and (20) are formulated as differential equations. Especially in the case of elasticity, it is often useful to switch to the integral form which means that, instead of velocities, displacements are considered (integral of velocities with respect to time). The corresponding equations are:

Translational:

$$\begin{aligned} \mathbf{s}_1 &= \mathbf{s}_0 + \mathbf{s}_2 + \Delta \mathbf{s} = \mathbf{s}_0 + \mathbf{s}_2 + \Delta \mathbf{s} \\ &= \mathbf{s}_0 + \mathbf{s}_2 + \mathbf{c}^{-1} \cdot \mathbf{F}_c = \mathbf{s}_0 + \mathbf{s}_2 + \mathbf{c}^{-1} \cdot \mathbf{F}_1 \end{aligned} \quad (21)$$

Rotational:

$$\begin{aligned} \varphi_1 &= \varphi_0 + \varphi_2 + \Delta \varphi = \varphi_0 + \varphi_2 + \Delta \varphi \\ &= \varphi_0 + \varphi_2 + \mathbf{c}_\varphi^{-1} \cdot \mathbf{M}_c = \varphi_0 + \varphi_2 + \mathbf{c}_\varphi^{-1} \cdot \mathbf{M}_1 \end{aligned} \quad (22)$$

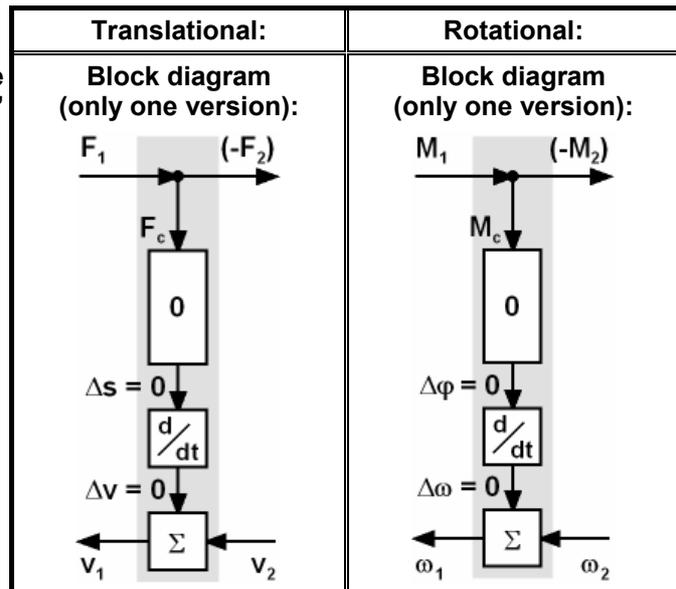
The values \mathbf{s}_0 or φ_0 in equations (21) and (22) are constants of integration (geometric distances or angles between the two ports to which the elastic [angular] deformation is added).

In reality, a linear element behaviour will be achieved with good approximation by appropriate design (e.g. "linear springs"). However, non-linear elastic effects also occur quite often – whether wanted or unwanted. In such cases the linear equations (19) and (21) have to be replaced by appropriate non-linear relations.

In systems theory, "cross-coupling elements" of the type displayed in figure 14 are called "C-elements" ("capacities"). Like the "I-elements" (inertias) described in the sub-section next to the last, also "C-elements" are not "power-conserving" but "energy-conserving".

Finally, also for the "cross-coupling elements" shown in Figure 14 there is a special case which is referred to quite often in the domain of mechanics: This is "force/torque transmission through a rigid body". This special case is formally defined by the condition $\mathbf{c}^{-1} = \mathbf{0}$ or $\mathbf{c}_\varphi^{-1} = \mathbf{0}$; the corresponding four-pole structure is shown in **figure 15**.

Figure 15. Special case
“rigid force/torque transmission”



4.4 Concluding remarks

Based on the fundamental four-pole element types introduced in the previous sections, it is now possible to build up multi-pole models of machine elements (and other components and systems, not necessarily confined to mechanical ones). For this purpose the basic element types will be coupled to one another according to the structure of the real system to be modelled.

In doing so, only the element types introduced can occur. However, especially in the field of mechanical elements and systems in very many cases non-linear equations will have to be used instead of the linear ones that were used to explain the concept of four-pole modelling. The equations which have to be used in a particular case result from the particular physical effect implied and from parameters which are defined by designing the system.

The equations to describe the (dynamic) system behaviour then result from “calculating backwards” through the whole system model, starting from the output values, following them up across all the different inner elements of the structure, until the input values are reached (as already shown schematically in figure 6).

Provided that elements of differentiation and/or integration occur in the “cross-couplings” we will end up with differential equations or integral equations, respectively, as descriptions of the transfer characteristics of the whole system.

5 Example “gear pair”

This chapter is supposed to demonstrate how multi-pole block diagrams are set up and how the resulting (differential) equations describing the (dynamic) system behaviour (transfer of input to output values) are deduced. The toothed gear pair, especially the spur gear variant, will be our example. This example is chosen because toothed gears belong to the most complex machine elements and because (for this reason?) their dynamic behaviour often is completely ignored – at least in basic teaching of machine elements.

In order to show that system modelling according to the principles and rules presented in this contribution can vary in level of details, we will make the multi-pole model of the gear pair gradually more complex by first ignoring and afterwards step by step adding effects and elements describing these. At the same time we want to show that the modelling method presented here can offer considerable advantages in understanding and explaining (the behaviour of) the machine element itself: The entire input/output behaviour of the element (including the dynamics) can be quite efficiently explained in terms of four-pole block diagrams and, based on these, (differential) equations describing this behaviour can be deduced; all this

can be carried on an abstract/formal level (and, subsequently, can be partly automated) without having to go into physical or even layout details.

For reasons of space, in this contribution the consideration will be confined to one set of spur gear pairs. In [Weber-05] some extended considerations are presented: cascaded gear pairs, gear trains with internal power-split (and the resulting question of load distribution) and epicyclic gear trains (having three external “port”/shafts).

5.1 Overview, first (“global”) approach

As an introduction **figure 16** shows the sketch of a spur gear pair to be examined. Apart from the input and output values at the two “ports” of the element ($M_1, \omega_1, M_2, \omega_2$), the sketch displays the parameters that contribute to the transfer of input to output values.

From a global point of view, the gear pair as a whole, according to **figure 16**, can be understood as a single element transmitting torque as well as angular velocity (thus transmitting rotational mechanical power), with the gear ratio i as the coefficient of the linear mathematical transfer function. The corresponding four-pole structure is shown in **figure 17**.

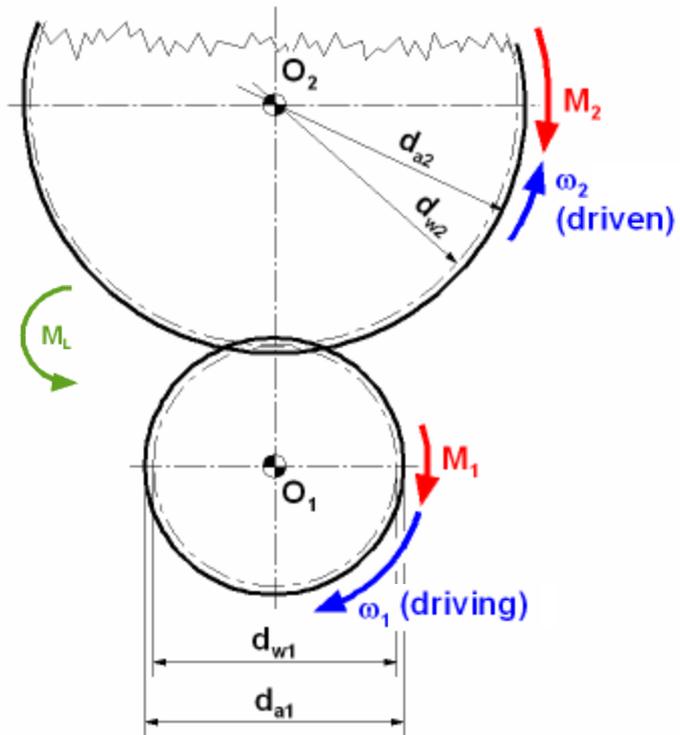
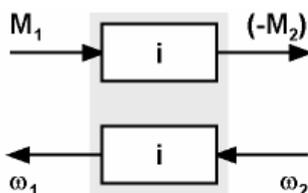


Figure 16. Toothed gear pair; sketch and functionally important dimensions →



← **Figure 17. Four-pole block diagram of toothed gear pair according to figure 16 (global view)**

The following transfer equations can be deduced from the block diagram:

$$\omega_1 = i \cdot \omega_2 \quad (23)$$

$$(-M_2) = i \cdot M_1 \quad (24)$$

The global approach may be sufficient if the gear pair is seen as a whole, e.g. as a component of a larger system. But if the functioning and the behaviour of the machine element itself is to be considered it is necessary to break down and examine more deeply its inner structure.

5.2 Rigid teeth, forces and motions considered in circumferential direction

As a first step of a more detailed consideration **figure 18** shows a sketch of the forces and velocities inside the gear pair in circumferential direction (u).

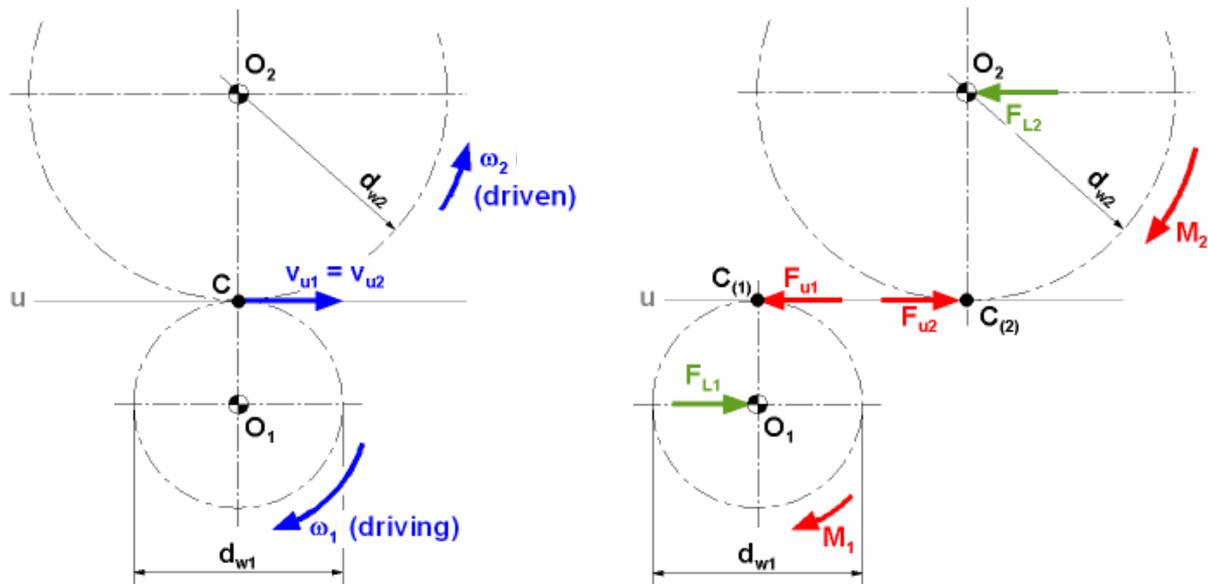


Figure 18. Forces and velocities inside the toothed gear pair according to figure 16 (circumferential consideration, teeth assumed rigid)

The related four-pole block diagram is shown in **figure 19**. It consists of two ideal transfer elements (converting the torques and angular velocities at the outer ports into circumferential forces and velocities by means of the two wheel bodies) plus a „cross-coupling element“. This element represents the force transmission between the two wheels, at present assumed to be rigid and in the case of gear wheels realised by positive drive through the teeth. The ideal transfer elements refer to section 4.1 (see figure 7), the „cross-coupling element“ is an example of elements describing elasticity effects discussed at the end of section 4.3, with the special case “rigid force/torque transmission” relevant here (see figure 15).

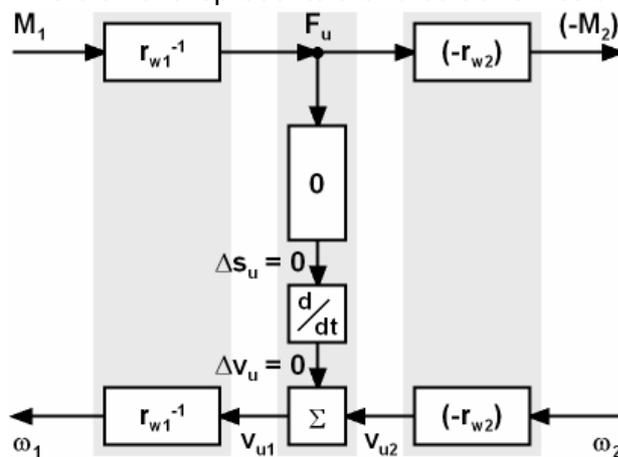


Figure 19. Four-pole block diagram of toothed gear pair (circumferential consideration, teeth assumed rigid)

The mathematical transfer functions that can be deduced from the block diagram according to figure 19 are the following:

$$\omega_1 = \frac{1}{r_{w1}} \cdot v_{u1} = \frac{1}{r_{w1}} \cdot (v_{u2} + \Delta v_u) = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 \quad (25)$$

$$(-M_2) = (-r_{w2}) \cdot F_u = -\frac{r_{w2}}{r_{w1}} \cdot M_1 \quad (26)$$

5.3 Elastic teeth, forces and motions considered in circumferential direction

It is now relatively easy to replace the assumption of a force transmission via a rigid pair of teeth with a force transmission via elastic teeth – bringing our considerations one step closer to reality. In doing so, we will at first stay with looking at forces and motions in circumferential direction.

With regard to this modification, **figure 20** shows a sketch of the forces (only); the velocities are the same as already shown in figure and not repeated here.

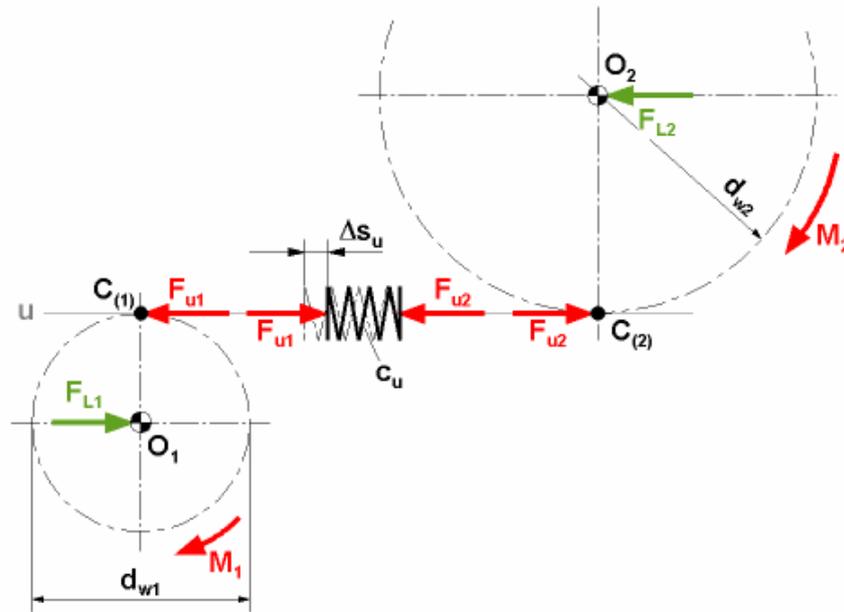
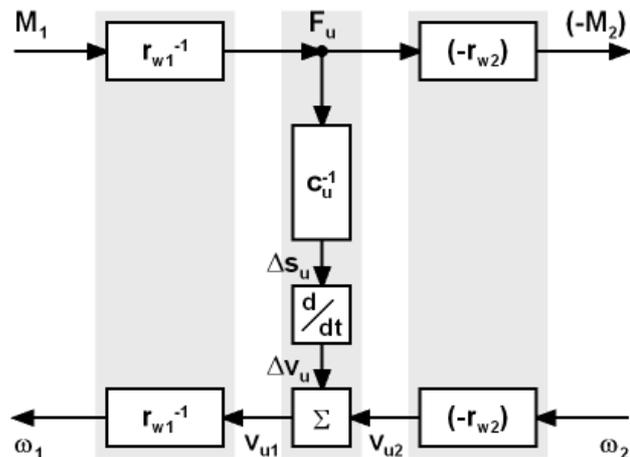


Figure 20. Forces inside the toothed gear pair, velocities as in figure 18 right (circumferential consideration, teeth assumed elastic)

It is important to notice that the transition from rigid to elastic teeth on the level of the four-pole block diagram is easily carried out simply by exchanging the “cross-coupling” of type “rigid force transmission” according to figure 15 for the more general one of the general “elasticity type” according to figure 14. The result is displayed in **figure 21**.

Figure 21. Four-pole block diagram of toothed gear pair (circumferential consideration, teeth assumed elastic)



The four-pole block diagram is now a means to deduce the transfer equations in a purely formal way. In order to do this, we gradually “calculate backwards” through the whole system model, starting from the output values (M_2 , ω_1), following them up across all the different inner elements of the structure, until the input values (M_1 , ω_2) are reached (see scheme in figure 6). Already in the still quite simple case considered here, this is much easier than analysing the behaviour by means of physical or layout details³:

³ In this as in all subsequent cases we will demonstrate how we establish the (differential) transfer equation by pointing out explicitly all the individual steps of the “calculating backwards” procedure throughout the four-pole structure.

$$\begin{aligned}\omega_1 &= \frac{1}{r_{w1}} \cdot v_{u1} = \frac{1}{r_{w1}} \cdot (v_{u1} + \Delta v_u) = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}} \cdot \frac{d}{dt}(\Delta s_u) \\ &= -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}} \cdot \frac{d}{dt} \left(\frac{1}{c_u} \cdot F_u \right) = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}^2} \cdot \frac{d}{dt} \left(\frac{M_1}{c_u} \right)\end{aligned}\quad (27)$$

$$(-M_2) = (-r_{w2}) \cdot F_u = -\frac{r_{w2}}{r_{w1}} \cdot M_1 \quad (28)$$

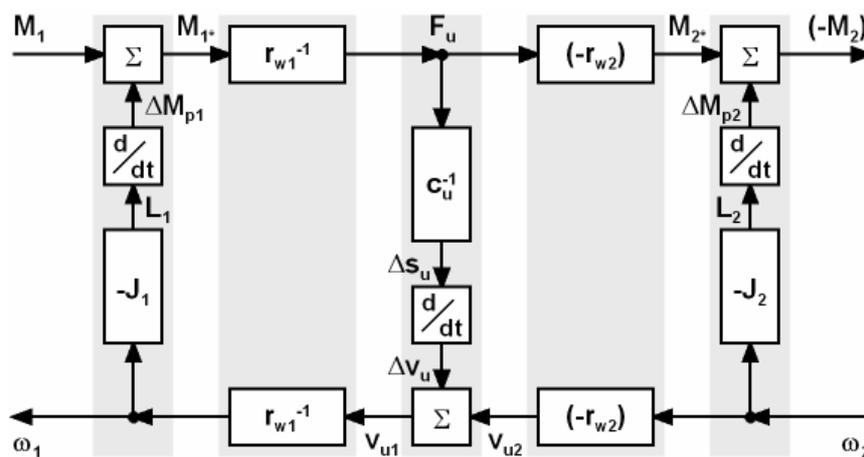
As the four-pole structure according to figure 21 for the first time in our study contains a differentiation element, one the resulting transfer equations is for the first time a differential equation (equation (27) for motion behaviour). The transfer equation (28) for forces/torques is identical to the one in the case of rigid teeth (see equation (26) in the last section 5.1). It is to be pointed out that the tooth spring rate c_u (or the compliance $\delta_u = c_u^{-1}$ as its reciprocal value) – both at present counted with respect to the circumferential direction – is **not** a constant but varies with the position of the point of contact and for this reason – taking motions into account – varies in time. Thus, the following term applies to the differential expression occurring in equation (27) even in cases where the driving torque M_1 is constant or is assumed to be constant in time:

$$\frac{d}{dt} \left(\frac{M_1}{c_u} \right) \neq 0$$

Therefore, the transfer of motion through a gear pair with elastic teeth is of non-constant velocity.

5.4 Elastic teeth plus taking the wheels' moment of inertia into account, forces and motions considered in circumferential direction

Analysing the problem in even more detail, it is now easy to take the (rotational) moments of inertia of the two wheel bodies additionally into consideration. For this purpose we integrate adequate “cross-couplings elements” into the four-pole structure (as explained in section 4.2, see figure 10). The resulting block diagram is shown in **figure 22**; it describes the behaviour



of a toothed gear pair with elastic teeth **and** rotational inertia effects, still with regard to the circumferential direction.

Figure 22. Four-pole block diagram of toothed gear pair (circumferential consideration, teeth assumed elastic, moments of inertia of the two wheel bodies considered)

In the present state of describing the (dynamic) behaviour of the toothed gear pair, the situation is already so complex that it would be quite tedious to deduce the transfer equation of the whole system by means of the physical model. In contrast to that, the method presented here (“calculating backwards” through the block diagram – which, of course, has to be there in the first place) can be carried out in a strictly schematic way, is simple and reliable:

$$\begin{aligned}
\omega_1 &= \frac{1}{r_{w1}} \cdot v_{u1} = \frac{1}{r_{w1}} \cdot (v_{u2} + \Delta v_u) = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}} \cdot \frac{d}{dt}(\Delta s_u) \\
&= -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}} \cdot \frac{d}{dt} \left(\frac{1}{c_u} \cdot F_u \right) = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}^2} \cdot \frac{d}{dt} \left(\frac{M_1^*}{c_u} \right) \\
&= -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}^2} \cdot \frac{d}{dt} \left(\frac{M_1 + \Delta M_{p1}}{c_u} \right) \\
&= -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}^2} \cdot \frac{d}{dt} \left(\frac{M_1 + \frac{d}{dt}(-J_1 \cdot \omega_1)}{c_u} \right)
\end{aligned} \tag{29}$$

$$\begin{aligned}
(-M_2) &= M_2^* + \Delta M_{p2} = (-r_{w2}) \cdot F_u + \frac{d}{dt}(J_2 \cdot \omega_2) \\
&= -\frac{r_{w2}}{r_{w1}} \cdot M_1^* + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= -\frac{r_{w2}}{r_{w1}} \cdot (M_1 + \Delta M_{p2}) + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= -\frac{r_{w2}}{r_{w1}} \cdot \left(M_1 + \frac{d}{dt}(-J_1 \cdot \omega_1) \right) + \frac{d}{dt}(-J_2 \cdot \omega_2)
\end{aligned} \tag{30}$$

Like in the case considered in the preceding section 5.3, the tooth spring rate c_u (or the compliance $\delta_u = c_u^{-1}$, respectively) – again counted with respect to the circumferential direction –, are **not** constant but depend on the position of the point of contact and for this reason – because of motions – also on time.

Equation (29) is the (differential) equation of motion. If you re-order it – all terms containing the output value ω_1 that has to be determined or any of its temporal derivations on one side – and assume that the rotational moments of inertia of the two wheels J_1 and J_2 are constant, the equation is the following:

$$\omega_1 + \frac{J_1}{r_{w1}^2} \cdot \frac{d}{dt} \left(\frac{1}{c_u} \right) \cdot \frac{d}{dt}(\omega_1) + \frac{J_1}{r_{w1}^2 \cdot c_u} \cdot \frac{d^2}{dt^2}(\omega_1) = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}^2} \cdot \frac{d}{dt} \left(\frac{M_1}{c_u} \right) \tag{31}$$

With the same premise ($J_1, J_2 = \text{const.}$) the equation of force can be re-formulated:

$$(-M_2) = -\frac{r_{w2}}{r_{w1}} \cdot M_1 - \frac{r_{w2}}{r_{w1}} \cdot J_1 \cdot \frac{d}{dt}(\omega_1) - J_2 \cdot \frac{d}{dt}(\omega_2) \tag{32}$$

The equation of motion (31) is an inhomogeneous, non-linear second order differential equation for ω_1 . It has to be solved at first; you may then insert the result $\omega_1 = f(\omega_2, M_1)$ into the equation of force (32) in order to calculate the result of the latter ($M_2 = f(\omega_2, M_1)$).

We do not go into the solution of the indicated differential equations in detail but will just present some general remarks about possible solution approaches:

- The most elegant way would be an analytical solution of the presented differential equations. In the case considered here (like in most cases relevant in engineering) this is, however, only possible by simplifying the equations considerably, sometimes up to an extent which completely distorts the physical reality.

In the present case, for instance, an often made assumption is $\omega_2 = \text{const.}$; this would immediately let disappear the last term of the right side of the force equation (32).

Another often made assumption is $\mathbf{M}_1 = \text{const.}$ But this alone does not help much because the last term on the right side of the velocity equation (31) still is not zero. But if, in addition to that, instead of a time-variable tooth spring rate \mathbf{c}_u an average constant tooth spring rate is substituted ($\bar{\mathbf{c}}_u$), then last term on the right side of equation (31) **plus** the middle term on the left side will equate zero. If you finally restrict the time variations of all time-dependent input and output values to sinusoidal forms, then equations (31) and (32) could be solved analytically. This solution, however, might only have little in common with reality.

- A second possibility to solve the equations analytically is to make use of the Laplace transformation (see for instance [Merz/Jaschek-03]) which in the field of control engineering is applied quite often. With this approach other than sinusoidal functions of time of input and output values can be analysed; these are, however, also ideal(ised) cases (e.g. step functions or Dirac's unit pulses at the external ports).
- If we want solutions without structural simplifications of the system and for arbitrary input/output functions, then we have to use computer support – quite comprehensive software is available for this today. The price to be paid is that a numerical solution always is a special solution which renders the special “system response” for concretely predetermined input value functions and system parameters (if these change in time).

5.5 Elastic teeth plus taking the wheels' moment of inertia into account, forces and motions considered in tooth-normal and tooth-tangential directions

The studies made so far still include an essential simplification: The forces and velocities transmitted between the teeth were analysed in circumferential direction. In reality, however, they are canted perpendicular (normal) to the teeth flanks, with additional forces and motions in tangential direction.

In order to satisfy the fundamental law of motion for toothed gears (see, for instance, [Niemann/Winter-03]) the direction normal to the teeth flanks differs from the direction of the pitch line in any point of contact \mathbf{X} (except in point \mathbf{C} where it could be, but does not have to be identical to the circumferential direction). The difference can be measured by the (local) pressure angle α_x . Note that in the general case (arbitrary kind of teeth geometry) the pressure angle α_x is **not** a constant parameter but a function of the momentary position of the contact point \mathbf{X} , i.e. α_x is, in the general case, time-dependent. Only the special case of involute toothing (which, however, is chosen in most applications) displays a constant pressure angle ($\alpha_x = \alpha_w = \text{const.}$) at every single contact point along the entire length of the contact.

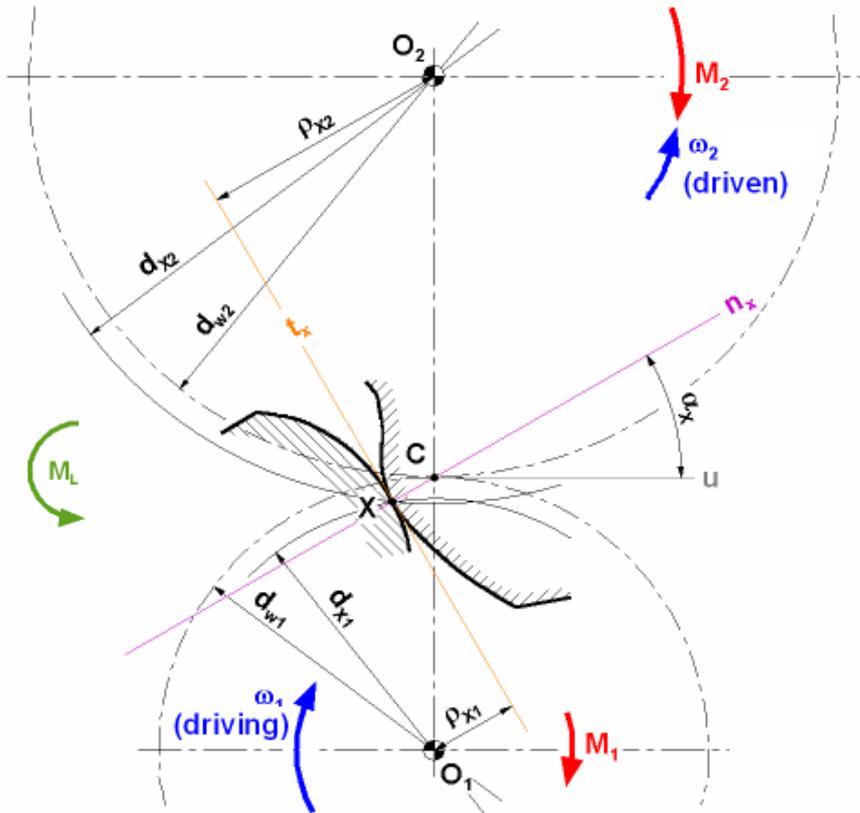
Figure 23 shows the sketch of a toothed gear pair modified accordingly. **Figure 24** explains forces and motions inside the gear pair in more detail – particularly how to resolve forces and velocities into components normal and tangential to the teeth (\mathbf{n} , \mathbf{t}) with contact in an arbitrarily chosen point \mathbf{X} .

The geometry parameters ρ_{x1} and ρ_{x2} are a measure of the momentary position of the contact point \mathbf{X} ; these parameters are therefore **not** constant over the line of contact. They are related by the following geometrical condition:

$$\frac{\rho_{x1} + \rho_{x2}}{r_{w1} + r_{w2}} = \sin \alpha_x \quad (33)$$

In order to keep the (differential) equations deduced further below relatively simple, the two geometry parameters ρ_{x1} and ρ_{x2} , which are in fact coupled by relation (33), will remain separate, despite the fact that one of them could be eliminated from all equations by applying equation (33) – but at the cost of much increased complexity.

Figure 23. Toothed gear pair; sketch and functionally important dimensions while having contact in point X



In the next paragraphs we will show in a step-by-step approach how to proceed – on the basis of four-pole modelling – from the previous simplified view with reference to the circumferential direction to an analysis with reference to directions normal and tangential to the teeth flanks. The starting point, representing the behaviour of a toothed gear pair with all forces and velocities expressed respect to the circumferential direction, is the four-pole block diagram already shown in the last section (figure 22).

At first we can observe that an elastic deformation of the teeth during the mesh now has to be related to the direction normal to the teeth flanks. Accordingly, the elastic behaviour has to be measured in normal direction, i.e. by the tooth spring rate c_n (or the compliance $\delta_n = c_n^{-1}$, respectively). Again, the tooth spring rate is *no* constant but varies with the position of the contact point and thus changes in time.

In addition to effects of inertia and elasticity, now also resistance/friction forces will be taken into consideration – this is the actual gain we get when proceeding from the previous simplified view to an analysis with respect to directions normal and tangential to the tooth flank.

In terms of the four-pole block diagram, this means that there have to be two separate branches inside the structure, both transmitting forces as well as velocities, one of them for forces and velocities in the direction normal to the teeth flanks, the other one for forces and velocities in tangential direction.

In order to draw up a revised four-pole model with one normal and one tangential branch, we have to integrate two junction elements – one for every tooth flank. At the two tooth flanks the forces sum up/are superimposed while every tooth flank certainly has to have a uniform velocity state. Therefore, the junction elements to be applied can only be of the type “summation/superposition of effort variables (here: forces), accompanied by identical flow variables (here: velocities) according to figure 9b in section 4.2.

The transmission of the force in the direction normal to the teeth flanks can be described by means of a “cross-coupling element” of the type “elasticity” – as has been used previously (see figures 21 and 22). Our reflections so far lead to the – still incomplete – four-pole block diagram shown in **figure 25, top**, as a preliminary result.

$$\begin{aligned}
(-M_2) &= M_2^* + \Delta M_{p2} = (-r_{w2}) \cdot F_{u2} + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= (-r_{w2}) \cdot [F_{u2n} + F_{u2t}] + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= (-r_{w2}) \cdot \left[F_{u1n} - \frac{\rho_{X2}}{r_{w2}} \cdot F_t \right] + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= (-r_{w2}) \cdot \left[(F_{u1} + F_{u1t}) - \frac{\rho_{X2}}{r_{w2}} \cdot F_t \right] + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= (-r_{w2}) \cdot \left[\left(\frac{M_1^*}{r_{w1}} + \frac{\rho_{X2}}{r_{w1}} \cdot F_t \right) - \frac{\rho_{X2}}{r_{w2}} \cdot F_t \right] + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= (-r_{w2}) \cdot \left[\left(\frac{1}{r_{w1}} \cdot [M_1 + \Delta M_{p1}] + \frac{\rho_{X2}}{r_{w1}} \cdot F_t \right) - \frac{\rho_{X2}}{r_{w2}} \cdot F_t \right] + \frac{d}{dt}(-J_2 \cdot \omega_2) \\
&= (-r_{w2}) \cdot \left[\left(\frac{1}{r_{w1}} \cdot \left[M_1 + \frac{d}{dt}(-J_1 \cdot \omega_1) \right] + \frac{\rho_{X1}}{r_{w1}} \cdot F_t \right) - \frac{\rho_{X2}}{r_{w2}} \cdot F_t \right] + \frac{d}{dt}(-J_2 \cdot \omega_2)
\end{aligned} \tag{35}$$

Equation (34) is the differential equation describing the behaviour of the motion values of the toothed gear pair according to figure 24 and the four-pole block diagram according to figure 25. Equation (35) is the corresponding (differential) equation to describe the force transfer behaviour.

Still missing in both equations (34) and (35) is a mathematical function for the resistance force at the tooth flanks ("law of friction"). In principle it looks like this:

$$\text{Law of friction: } \mathbf{F}_t = \mathbf{f}(\Delta \mathbf{v}_t) \tag{36}$$

$\Delta \mathbf{v}_t$ is the relative velocity between the teeth in tangential direction ("relative slip"). It can also be determined from the four-pole block diagram according to figure 25 by "calculating backwards", this time starting with $\Delta \mathbf{v}_t$:

$$\begin{aligned}
\Delta \mathbf{v}_t &= \mathbf{v}_{t1} + \mathbf{v}_{t2} = \frac{\rho_{X1}}{r_{w1}} \cdot \mathbf{v}_{u1} - \frac{\rho_{X2}}{r_{w2}} \cdot \mathbf{v}_{u2} = \frac{\rho_{X1}}{r_{w1}} \cdot \frac{1}{\cos \alpha_X} \cdot \mathbf{v}_{n1} - \frac{\rho_{X2}}{r_{w2}} \cdot \mathbf{v}_{u2} \\
&= \frac{\rho_{X1}}{r_{w1}} \cdot \frac{1}{\cos \alpha_X} \cdot (\mathbf{v}_{n2} + \Delta \mathbf{v}_n) - \frac{\rho_{X2}}{r_{w2}} \cdot \mathbf{v}_{u2} \\
&= \frac{\rho_{X1}}{r_{w1}} \cdot \mathbf{v}_{u2} + \frac{\rho_{X1}}{r_{w1}} \cdot \frac{1}{\cos \alpha_X} \cdot \frac{d}{dt} \left(\frac{\mathbf{F}_n}{c_n} \right) - \frac{\rho_{X2}}{r_{w2}} \cdot \mathbf{v}_{u2} \\
&= \left[\frac{\rho_{X1}}{r_{w1}} - \frac{\rho_{X2}}{r_{w2}} \right] \cdot \mathbf{v}_{u2} + \frac{\rho_{X1}}{r_{w1}^2} \cdot \frac{1}{\cos \alpha_X} \cdot \frac{d}{dt} \left(\frac{r_{w1} \cdot \mathbf{F}_{u1} + \rho_{X1} \cdot \mathbf{F}_t}{c_n \cdot \cos \alpha_X} \right) \\
&= (-r_{w2}) \cdot \left[\frac{\rho_{X1}}{r_{w1}} - \frac{\rho_{X2}}{r_{w2}} \right] \cdot \omega_2 + \frac{\rho_{X1}}{r_{w1}^2 \cdot \cos \alpha_X} \cdot \frac{d}{dt} \left(\frac{M_1 + \frac{d}{dt}(-J_1 \cdot \omega_1) + \rho_{X1} \cdot \mathbf{F}_t}{c_n \cdot \cos \alpha_X} \right)
\end{aligned} \tag{37}$$

In equation (37) you can see as first term the familiar result arising from a purely steady-state analysis (see, for instance, [Niemann/Winter-03]). But in the dynamic case discussed here, a second term is added that considers superimposed influences of outer and inner values which are generally not constant in time.

In terms of solving our problem as a whole, the results are not very pleasant: In principle you would have first to solve the (differential) equation of motion (34) for the gear pair as a whole before you can satisfy equation (37). Equation (37), however, was only introduced to insert its solution into equation (36) which, in turn, is needed to solve the original equation (34). Only a (computer-supported) iterative approach could solve the while problem, but this shall not be discussed in detail here.

In order to determine the resistance/friction force F_t with respect to the relative tangential velocity Δv_t (equation (36)) in the most simple (albeit often used) case an approach according to Coulomb (“dry”) friction is applied [Niemann/Winter-03]. In this case the dependence of Δv_t would have the mathematical form of a signum function. Conditions are getting more and more complex if we take Newton’s law of fluid friction or mixed dry/fluid friction as a basis. In this contribution, the main focus of which is not the exploration of friction in toothed gears, we will not elaborate on this subject.

If we now assume – as already done before – that the two moments of inertia do not change in time ($J_1, J_2 = \text{const.}$) and if we re-order equations (34) and (35) in such a way that all terms containing the respective output value (ω_1, M_2) or derivatives of it are written on one side we obtain the following formulation:

$$\begin{aligned} \omega_1 - \frac{d/dt(c_n \cdot \cos \alpha_X)}{r_{w1}^2 \cdot c_n^2 \cdot (\cos \alpha_X)^3} \cdot J_1 \cdot \frac{d}{dt}(\omega_1) + \frac{1}{r_{w1}^2 \cdot c_n \cdot (\cos \alpha_X)^2} \cdot J_1 \cdot \frac{d^2}{dt^2}(\omega_1) \\ = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 - \frac{d/dt(c_n \cdot \cos \alpha_X)}{r_{w1}^2 \cdot c_n^2 \cdot (\cos \alpha_X)^3} \cdot (M_1 + \rho_{X1} \cdot F_t) + \frac{d/dt(M_1 + \rho_{X1} \cdot F_t)}{r_{w1}^2 \cdot c_n \cdot (\cos \alpha_X)^2} \end{aligned} \quad (38)$$

$$(-M_2) = -\frac{r_{w2}}{r_{w1}} \cdot M_1 - \frac{r_{w2}}{r_{w1}} \cdot J_1 \cdot \frac{d}{dt}(\omega_1) - r_{w2} \cdot \left[\frac{\rho_{X1}}{r_{w1}} - \frac{\rho_{X2}}{r_{w2}} \right] \cdot F_t - J_2 \cdot \frac{d}{dt}(\omega_2) \quad (39)$$

As was already mentioned above in the (often used) special case of involute tothing the following applies:

$$\text{Involute tothing: } \alpha_X = \alpha_w = \text{const.} \quad (40)$$

Using equation (40) for gear pairs with involute tothing the (differential) equation (38) of motion can be substantially simplified:

$$\begin{aligned} \omega_1 + \frac{J_1}{r_{w1}^2 \cdot \cos^2 \alpha_w} \cdot \frac{d}{dt} \left(\frac{1}{c_n} \right) \cdot \frac{d}{dt}(\omega_1) + \frac{J_1}{r_{w1}^2 \cdot \cos^2 \alpha_w \cdot c_n} \cdot \frac{d^2}{dt^2}(\omega_1) \\ = -\frac{r_{w2}}{r_{w1}} \cdot \omega_2 + \frac{1}{r_{w1}^2 \cdot \cos^2 \alpha_w} \cdot \left[\frac{d}{dt} \left(\frac{M_1}{c_n} \right) + \frac{d}{dt} \left(\frac{\rho_{X1} \cdot F_t}{c_n} \right) \right] \end{aligned} \quad (41)$$

Again we will not elaborate on the solution of the differential equation.

5.6 Concluding remarks

The example of toothed gears could be extended a lot further. As was already stated at the beginning of this chapter, in [Weber-05] some additional studies on cascaded gear pairs, gear trains with internal power-split (and the resulting question of load distribution) and epicyclic gear trains (having three external “port”/shafts) are presented. Other extensions would be helical gears, bevel wheels, etc. All this will be stopped now, because the main task of the example was to demonstrate how the principles and methods of setting up and analysing multi-pole block diagrams elucidated in chapter 4 can be applied in the field of mechanical components, including the field of machine elements. The aim was to demonstrate

- that on the basis of a given multi-pole block diagram the equations describing the (dynamic) transfer behaviour of a system can be derived following to strictly formal, basically simple criteria which in complex cases is a lot easier than analyses based on physical or layout models,
- that the results identical, sometimes even more accurate than those attained by a conventional analysis because simplifications do not have to be implemented a priori,

- that the setting up of four-pole block diagrams, which are the precondition for the approach shown, is itself realised according to relatively simple principles and rules, and
- that multi-pole models for mechanical (and other) components are “scaleable”, i.e. they can be adapted to different development phases and/or application scenarios by adding or leaving out effects (from a very simplified “global” up to a most detailed study)⁴.

The only precondition to use the principles and methods demonstrated in this paper is to get familiar with the logical/functional way of reasoning. However, the author considers the relative abstractness of the approach to be an advantage rather than a problem, because it is necessary for the comprehensive dealing of mechatronic systems anyway.

6 Summary and conclusions

This paper has attempted to expose how to develop and use models of mechanical components on the basis of the multi-pole theory. The author’s recapitulating conclusion is that these models are readily applicable in this field, can provide some advantages and could be a possible unified basis for the modelling (and subsequent simulation) of mechatronic systems.

When asking for alternatives, on one hand control block diagrams [Isermann-99, Nordmann/Birkhofer-03], and on the other hand so-called Bond Graphs [Wellstead-79, Breedveld-84, Karnopp et al-00] are named and sometimes clearly favoured. In [Weber-05] advantages and disadvantages of these alternatives are briefly discussed and reasons are given for the author’s preference of the four-pole models presented here.

The introduction of multi-pole block diagrams for mechanical (and other) components as well as the subsequent derivation of the transfer (differential) equations are not an end in itself, but precondition and basis for computer-supported system simulation. For that purpose, the equations/equation systems have to be transferred into corresponding programmes (like MatLab Simulink), here they have often to be re-formulated and can then be (numerically) solved. This is not always possible without problems, but these shall not be discussed here (see [Weber-05] which contains at least some basic information).

To conclude, a couple of notes concerning teaching issues in the field of machine elements should be made:

The author is convinced that machine elements teaching has to move towards a more systematic approach based on design science or design theory and methodology [Eder-04, Eder-05a, Eder-05b], and at the same time has to enhance the scope towards mechatronic elements and systems (as was already presaged in the so-called Heiligenberg Manifesto in 1997 [AKME-97]). To achieve this, first steps have already been taken at several universities – above all at the Technical University of Darmstadt [Nordmann/Birkhofer-03]). Also at Saarland University, the author’s home institution, the machine elements course was extensively reorganised: Since winter term 2004/2005 it carries the new title “mechatronic elements and systems” (similar to Darmstadt) and is (again similar to Darmstadt) jointly lectured by the author and a colleague with a mechatronics and systems theory/simulation background (Prof. Andreas Kugi). In these lectures, parts of the principles and methods of modelling presented in this paper are already used (together with a thorough physical foundation of the subject which was not addressed in this paper).

Of course, this costs additional effort and extra time. So what are the benefits?

In the author’s opinion they are not at all confined to making machine elements “mechatronics-compatible”. Even more important could be benefits for the subject “machine elements” itself: The logical/functional way of reasoning opens new options to explain the functioning and the physics of machine elements much more systematically and sometimes even more accurately (i.e. with less simplifying assumptions and with more overview) than is possible with traditional approaches and methods.

⁴ This was, in fact, only sketchily shown in this paper because of the bottom-up approach (proceeding from simpler to more complex models). There are relatively simple rules for the systematic top-down simplification of multi-pole structures which are not presented here for lack of space.

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