# RELATING THE SENSITIVITY OF PACK ASSEMBLY PROCESSES TO MACHINE CONFIGURATION AND PACK GEOMETRY 

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#### Abstract

In many industrial applications, particularly those within the packaging industry, products and packs are moved around using various conveying systems. There is frequently a need to rearrange items into different configurations in order to achieve required flexibility, attain speeds of operations, and perform specific tasks. This paper looks at the issue of "single laning" in which packs that span across a conveyor are moved into a single file. The problems encountered in this area are discussed and the sensitivity of current designs to bridging effects are investigated. This investigation is combined with theoretical studies to generate design rules that provide the basis for developing improved machine configurations and processes.


Keywords: Industrial applications, industrial case study, introduction of methods in industry

## 1 Introduction

The packaging machinery industry like many other sectors of manufacturing is a highly competitive global market, driven by the ever-increasing requirements of the customer. These demands include the ability to handle a greater variation in product sizes and characteristics with increased production rates [1]. In order to meet these demands with existing systems and machine designs, many manufacturers fine-tune assembly processes for a specific product and certain operating conditions [2]. This trial-and-error approach is time-consuming and may well frustrate the development process of the next generation of machines, their performance capabilities and ultimately their commercial success [3].

In order for the manufacturers of packaging machinery to remain competitive in the future it is necessary to develop a fundamental understanding of the interactions that occur between pack and assembly operation. From this understanding it is then possible to evaluate the sensitivity of this interaction to changes in pack geometry or attributes. In this manner, improved packing systems can be created which reduce this sensitivity to variations and either attenuate or even eliminate the configurations and conditions that limit performance. This paper presents an investigation into the process of single laning and the phenomenon of 'bridging', which causes production to halt and often results in significant downtime.

### 1.1 Single laning

The process of single laning involves rearranging or organising multiple packs on a production line into one or more discrete lanes, so that subsequent operations may be carried out on individual packs. Single laning is commonplace in many sectors of the packaging industry and is often a prerequisite for washing, filling, capping and labelling of bottles or containers. The process of single laning may be undertaken by a variety of different mechanisms. These include the use of complex guides, servo-controlled single and multiple conveyors, compressed air, rotary accumulators and vibrating beds. As performance demands
escalate and the desire for systems to be capable of handling packs over a wider range of attributes increases, the numbers of observed failures and downtime have risen. This rise is primarily due to the greater number of occurrences of the conditions or configurations where failure may occur. These conditions represent the limits of the acceptable performance bounds of a process. Such limits depend upon the attributes of the pack and the assembly process. For the problem of single laning, the phenomenon of "bridging" is seen when these limits are exceeded, or where certain critical values arise. Bridging occurs when a number of packs appear to lock together across the transport system. This halts product flow and production, and usually requires operator intervention. Figure 1 shows an example machine configuration for a single laning operation. The input region of the single laning process provides an accumulation zone that is fed either by conveyors from previous process machines or the placing of product from mass pack containers. It is the speed of passage through the single laning process that determines whether the accumulation zone either empties or backs up.


Figure 1. Examples of bridging in a convergence zone.

### 1.2 Bridging

The main cause of bridging is the creation of locking structures of product between the side walls and guides on the conveyor system. The corralling of a group of products into a single file results in relative movement within the group that can on occasions form strong locking bridges across the direction of product flow that can halt the complete plant.
In the simplest approach to creating a single lane output, a convergence zone is employed (shown in Figure 1). On the right hand side packs of basically rectangular forms are shown forming a strong bridge. Such an inadvertent structure cannot easily be broken up particularly when the upstream accumulation zone is full of waiting product. It is for this reason that the majority of rectangular packs are handled by discrete transfer mechanisms rather than by deflecting sidewalls.
The main use of converging channels occurs for circular based packs (such as bottles) where the interaction of the product is less likely to create bridges. These can still however occur as shown in the left hand portion of figure 1.


Figure 2. Arch formed from cylindrical components.
Bridges occur when the packs form a compression arch between fixed boundary points. The arch is only stable if the reaction forces between the arching packs are balanced and the reaction forces between them lie within the angle of their coefficient of friction (Figure 2). Whilst perfectly friction-free objects can only form arches if the ideal arch is accidentally formed, most normal cylindrical packs form arches over a relatively wide variation in arch shape.

### 1.3 Current approaches

Many practical approaches have been applied to the solution of the problem of bridging. These operate on a combination of differing basic principles as follows.

- Vibrating elements to break up bridges that form
- Multiple conveyor belts that move the various rows of products at differing speeds to create gaps into which adjacent product can merge
- Air jets to move and manipulate packs into a desired order
- Vacuum belts with pack location features
- Rotary accumulation tables
- Alignment rails
- Flow interrupting devices to provide traffic control of packs

These approaches all attempt to create differential velocities of the product in the convergence zone in order to allow gaps to be formed. These gaps permit the product across the conveyor to merge into a single continuous line in the direction of product flow. In general, these approaches attempt to reduce the likelihood of a pack configuration where bridging may occur and if it occurs some approaches attempt to disturb the packs to destroy the bridge. However, many of these current designs still exhibit what appear to be random failures. In order to address this, the work in this paper investigates the geometric sensitivity of a particular pack and configuration of convergent guides to bridging. The results of this investigation are used to develop a revised configuration of guides where bridging is less likely to occur.


Figure 3. Modelling of forces within an irregular shaped pack.

## 2 Theoretical modelling and experimental studies

In order to support the development of a strategy for the design and improvement of single laning devices, a number of theoretical models have been created and experimental investigations undertaken. The movement of packs against fixed side walls, dividers and against adjacent packs was undertaken in a separate study [2]. Here mathematical models were created of the interaction of random shaped objects and attempts made to predict the final directions of motion through the balancing of energies (Figure 3).


Figure 4. Theoretical models.
Further models were created within a constraint modelling environment to investigate the movements of cylindrical packs during the process of realignment into a single lane. A number of the standard approaches were modelled and their problems investigated (Figure 4). All of these were further investigated and compared with a physical system. For this, an experimental rig was created in which the motions of cylindrical objects were recorded with a high-speed digital camera (Figure 5). The analysis of the experimental results showed good
correlation to the theoretical models. The results of the theoretical modelling were then used to provide the basic understanding and rules for techniques to allow existing single laning devices to be improved and from which new approaches could be created. In particular, the combination of theoretical modelling and experimental investigation provides an understanding of the phenomenon of bridging and enables the generation of design rules for the design and synthesis of machines that avoid or significantly reduce the likelihood of bridging.


Figure 5. Experimental rig for the investigation of pack movement on a conveyor.

### 2.1 Design rules

A number of simple but fundamental rules emerged from these studies that control the working and effectiveness of single laning devices.

For the device not to block, or back up product into previous processing stations, the mass flow rate on exit must at least equal that of the incoming. This requires that, if the number of packs across the conveyor width is $N$, then the output velocity must be greater than $N$ times that of the incoming product. Whilst the increase in belt speed is only required on exit to satisfy the above requirement, an increase in speed is required early in the convergence zone in order to create space between packs so that they can be moved into alignment.
The experimental investigation of bridging structures revealed that bridges generally form at particular locations along a converging section of guide rails. If this location can be determined, then mechanisms or methods to either eliminate these critical points or to
incorporate devices to break up bridges when they occur can be developed. In this work, the approach adopted is to attempt to eliminate the critical points along a convergent guide configuration where bridging may occur. The following section discusses a mathematical model of bridging for the identification of the regions along a set of convergent guides where bridges are likely to occur.

## 3 Mathematical model of product bridging in convergent guides

Bridging can occur when products are moving along a conveyor in a region where the sides of that conveyor start to converge. The effect is that the products lock together so as to prevent themselves moving. The model looks at a very simple case of bridging. This is for objects with circular cross section and with any frictional effects ignored. A simple relation between the angles involved is found to exist.

Consider the case of $n$ circular objects bridging. They are numbered $1,2, \ldots, n$. An example in the case $n=5$ is shown in figure 6 . There are ( $n-1$ ) points of contact between the objects themselves, and two more between the objects and the converging guides. Where objects $i$ and $(i+1)$ touch, the normal to their common tangent passes through the centres of both circles. Let $\alpha_{i}$ denote the angle between the common tangent and the direction of motion of the conveyor. This is also the angle between the line joining the centres and the direction perpendicular to motion of the conveyor. Similarly, let $\alpha_{0}$ and $\alpha_{n}$ be the angles between the guides and the direction of conveyor. Both of these are known. Note that it is not assumed initially that the arrangement is symmetric; there is no need for $\alpha_{0}$ and $\alpha_{n}$ to be equal.
For convenience, set

$$
s_{i}=\sin \alpha_{i} \quad c_{i}=\cos \alpha_{i} \quad t_{i}=\tan \alpha_{i}
$$

Let $R_{i}$ denote the normal reaction between objects $i$ and $(i+1)$. Let $R_{0}$ and $R_{n}$ denote the normal reactions at the guides. Use $P$ to denote the (frictional) force imposed on each body by the conveyor moving underneath.


Figure 6. Definition of the angles.

### 3.1 Assumptions

In this work a number of assumptions are made. Firstly, any frictional effects are ignored between the objects themselves and between the objects and the guides. Secondly, we assume that the frictional force from the conveyor is assumed to be the same on each object and acts in the direction of the movement of the conveyor. Thirdly, it is assumed that the only forces acting on the objects are those identified above, namely the friction from the conveyor and the reactions between the bodies themselves and with the guides. Forces from any other bodies up-stream of the bridge are ignored.

Note that, although the figures show the objects having the same radius, this assumption is not needed until it is necessary to try to find where the bridge occurs.


Figure 7. Forces on adjacent objects.

### 3.2 Relation between the angles

Consider the arrangement shown in figure 7 where objects $i,(i+1)$, and $(i+2)$ come together. Assume that the forces on object $i$ are in equilibrium and resolve them in the direction of the conveyor and perpendicular to it.

$$
\begin{align*}
& R_{i-1} \sin \alpha_{i-1}-R_{i} \sin \alpha_{i}=P  \tag{1}\\
& R_{i-1} \cos \alpha_{i-1}-R_{i} \cos \alpha_{i}=0 \tag{2}
\end{align*}
$$

Taking $\cos \alpha_{i}$ times equation (1) and $\sin \alpha_{i}$ times equation (2) and subtracting, gives the following.

$$
\begin{equation*}
R_{i-1} \sin \left(\alpha_{i-1}-\alpha_{i}\right)=P \cos \alpha_{i} \tag{3}
\end{equation*}
$$

Similarly, taking $\cos \alpha_{i-1}$ times equation (1) and $\sin \alpha_{i-1}$ times equation (2) and subtracting, leads to

$$
\begin{equation*}
R_{i} \sin \left(\alpha_{i-1}-\alpha_{i}\right)=P \cos \alpha_{i-1} \tag{4}
\end{equation*}
$$

Replacing $i$ by ( $i+1$ ) in equation (3), yields

$$
\begin{equation*}
R_{i} \sin \left(\alpha_{i}-\alpha_{i+1}\right)=P \cos \alpha_{i+1} \tag{5}
\end{equation*}
$$

Dividing equation (4) by equation (5) shows that

$$
\begin{equation*}
\frac{R_{i} \sin \left(\alpha_{i-1}-\alpha_{i}\right)}{R_{i} \sin \left(\alpha_{i}-\alpha_{i+1}\right)}=\frac{P \cos \alpha_{i-1}}{P \cos \alpha_{i+1}} \tag{6}
\end{equation*}
$$

Multiplying across and expanding the sines gives the following expression

$$
s_{i-1} c_{i} c_{i+1}-c_{i-1} s_{i} c_{i+1}=c_{i-1} s_{i} c_{i+1}-c_{i-1} c_{i} s
$$

When $c_{i}=\cos \left(\alpha_{i}\right)$ and $s_{i}=\sin \left(\alpha_{i}\right)$. Dividing throughout by $c_{i-1} c_{i} c_{i+1}$ and rearranging provides a relation between the tangents of the angles, when $t_{i}=\tan \left(\alpha_{i}\right)$

$$
\begin{equation*}
t_{i+1}-2 t_{i}+t_{i-1}=0 \tag{6}
\end{equation*}
$$

This is a difference equation whose characteristic polynomial

$$
p^{2}-2 p+1
$$

has a repeated root of 1 . Hence the general solution of equation (6) is

$$
\begin{equation*}
\tan \alpha_{i}=t_{i}=A+B i \tag{7}
\end{equation*}
$$

where $A$ and $B$ are constants. Note that no assumption has been made that the objects have the same radius.

### 3.3 Symmetric case

From now on, assume that the guides are arranged symmetrically about the conveyor. Let $\alpha_{0}=$ $\beta$. Then, allowing for an implicit sign convention, symmetry gives $\alpha_{n}=-\beta$. The constants in equation (7) can now be found. Substitution shows that

$$
\begin{gathered}
\tan \beta=A \\
-\tan \beta=t_{i}=A+n B
\end{gathered}
$$

The latter shows that $B=-2 A / n$ and so

$$
\begin{equation*}
\tan \alpha_{i}=t_{i}=\left[\frac{n-2 i}{n}\right] \tan \beta \tag{8}
\end{equation*}
$$

Set $S=\sin \beta$ and $C=\cos \beta$. Then the last equation becomes

$$
t_{i}=\left[\frac{(n-2 i) S}{n C}\right]
$$

The sum of the squares of the numerator and denominator here are

$$
(n-2 i)^{2} S^{2}+n^{2} C^{2}=(n-2 i)^{2}\left(1-C^{2}\right)+n^{2} C^{2}=(n-2 i)^{2}+4 i(n-i) C
$$

Hence, it follows that

$$
\begin{equation*}
\cos \alpha_{i}=c_{i}=\frac{n c}{\left.\sqrt{\left[(n-2 i)^{2}+4 i(n-i) C\right.}\right]} \tag{9}
\end{equation*}
$$

It is now possible to find where the bridge occurs in the symmetric case. Suppose the line joining the contacts with the guides has length $w$. Suppose also that this line is distance $d$ from the point where the guides would intersect. These are shown in figure 8 .
These are connected by

$$
\begin{equation*}
\tan \beta=\frac{w}{2 d} \tag{10}
\end{equation*}
$$

Now assume that the objects all have the same radius $r$. So far, no assumption about the radii has been made. In figure 6 , the distance between the centres of objects $I$ and $(i+1)$ in the direction perpendicular to the conveyor is

$$
2 r \cos \alpha_{i+1}=2 r c_{i+1}
$$

Hence, taking allowance for what happens at the ends of the bridge, we deduce that

$$
\begin{equation*}
w=r\left[c_{0}+2 c_{1}+2 c_{2}+2 c_{3} \ldots+2 c_{n-3}+2 c_{n-2}+2 c_{n-1}+c_{n}\right] \tag{11}
\end{equation*}
$$

Taking equations (9), (10) and (11) together, allows $d$ to be found and so determine where the bridge occurs.


Figure 8. Location of a bridge.
In the case $n=5$, the cosines are as follows.

$$
\left.\left.c_{0}=c_{5}=C, c_{1}=c_{4}=5 C / \sqrt{9+16 C^{2}}\right], c_{2}=c_{3}=5 C / \sqrt{1+24 C^{2}}\right]
$$

For example, if we take $\beta=30^{\circ}$, then $C=0.8660$ and the following is the table of values for the case $n=5$.

| $i$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $c_{i}$ | 0.8660 | 0.9449 | 0.9934 | 0.9934 | 0.9449 | 0.8660 |
| $\alpha_{i}$ | 30.00 | 19.11 | 6.59 | -6.59 | -19.11 | -30.00 |

Figure 9. Cosine values for case $n=5$.
Then $w=9.485 r$ and $d=8.214 r$. Figure 8 is drawn using these values.
The mathematical modelling of these stable structures shows that arches for a given pack size can only be formed at discrete positions in the convergence region for given channel convergence angles. Whilst these can theoretically be formed for any number of packs (up to the width of the complete belt) the most critical ones are the lowest numbers of 2, 3, and possibly 4 as these are stable over the widest angle of product interaction. Higher arches are normally broken up as they form due to the random movement between products as they move down the conveyor belt. The positions at which these primary bridging structures can occur, for a given convergence angle and cylinder size, can be calculated.

## 4 New approach

As previously discussed, the design rules can be used to either correct errors in existing designs, or to aid the creation of new approaches for improved machine and process design. In this work, the design rules are used to facilitate a new configuration for single laning using converging guides. The sensitivity of convergent sections of guides to pack geometry and the phenomenon of bridging are explored.
Through the creation of a mathematical model of bridging structures it is possible to identify for a particular pack and configuration of guides the regions where bridging is likely to occur. Having identified these locations a strategy was developed which removes these known bridging points. This strategy involves breaking the convergent guides at each of the known bridging points and redirecting product flow towards the opposite side of the conveyor. This
is shown in Figure 10. The regions of potential bridging were thus disturbed in order to prevent such blockages occurring. The breaking up of bridges can then be discouraged by subtle changes in geometry or the inclusion of actively moving devices in these regions to disturb the foundations of the arch structure.


Figure 10. Zigzagging track to break up bridging.

## 5 Conclusions

The study has shown that through combined experimental and practical studies the sensitivity of an existing process to pack geometry and attributes can be modelled. The results of this modelling can then be used to improve existing machine design and set up as well as provide a set of design rules for the next generation of machines and processes. The latter of these is essential if machinery manufacturers are to compete at an international level. This work deals with the process of single laning and in particular, the problem of product bridging across convergent sections of conveyors. A practical investigation and theoretical study of current design principles is undertaken. This provides an understating of the process and allows the identification of design rules. A simple mathematical model of bridging has been created and used to evaluate the sensitivity of a particular pack and guide configuration to bridging. These design rules and the results of the mathematical model are used to develop a new approach and a revised conveyor configuration which is less sensitive to the phenomenon of bridging.

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